Algebra 1: Principles of Secondary Mathematics
Bridge Material

Below are a few resources to help you and your student prepare for algebra. Use any combination of these, or other materials you identify, to review the prerequisite skills noted on your student's Readiness Check.

Fractions

Prerequisite Skill: 3, 4, 5

- Prior Math-U-See levels
    - Epsilon
        - Adding Fractions (Lessons 5, 8)
        - Subtracting Fractions (Lesson 5)
        - Multiplying Fractions (Lesson 9)
        - Dividing Fractions (Lesson 10)
        - Simplifying Fractions (Lessons 12, 13)
        - Recording Mixed Numbers as Improper Fractions (Lesson 15)
        - Mixed Numbers (Lessons 17–25)
    - Pre-Algebra
        - Fractional Exponents (Lesson 5)
- Objectives/Skills
    - Write a fraction in simplest form.
    - Write mixed numbers as improper fractions.
    - Perform all operations on fractions, including mixed numbers: adding, subtracting, multiplying, dividing, and raising to a power
- Additional Practice
    - Math-U-See Worksheet Generator located in the Digital Toolbox

Operations with Fractions

When performing operations with fractions, keep the following in mind:

- Fractions must have the same denominator in order to add or subtract.
- Remember to write all mixed numbers as improper fractions BEFORE performing any operations.
- Fractions do not need to be written as mixed numbers but do need to be simplified.
- When the directions say "evaluate," perform the given operation on the fraction and write it in simplest form.

Example 1

Evaluate.

Implement	**Explain**
$\frac{4}{7} - \left(-\frac{1}{3}\right)$	◀ Subtracting a negative makes a positive, so add the fractions.
$\frac{4}{7} + \frac{1}{3}$	◀ The LCD (lowest common denominator) for 7 and 3 is 21. Use the rule of 4 (cross-multiply) to give the fractions the same denominator of 21.
$\frac{12}{21} + \frac{7}{21} = \frac{19}{21}$	◀ When adding or subtracting fractions, only add or subtract the numerators. The denominator stays the same.

Example 2

Evaluate.

Implement	Explain
$\frac{23}{4} - \frac{5}{6}$	Before performing the operation, change the mixed number into an improper fraction.
$\frac{11}{4} - \frac{5}{6}$	The LCD for 4 and 6 is 12. Multiply the first fraction by $\frac{3}{3}$ and the second fraction by $\frac{2}{2}$. This gives both fractions a new denominator of 12.
$\frac{33}{12} - \frac{10}{12} = \frac{23}{12}$	The final answer can be left as an improper fraction.

Example 3

Evaluate.

Implement	Explain
$-3\frac{2}{5} \div 1\frac{2}{7}$	First, change the mixed numbers into improper fractions.
$-\frac{17}{5} \div \frac{9}{7}$	Multiply the first fraction by the reciprocal of the second fraction.
$-\frac{17}{5} \cdot \frac{7}{9}$	There are no common factors to simplify, so multiply straight across the numerators and denominators.
$-\frac{17}{5} \cdot \frac{7}{9} = -\frac{119}{45}$	

Example 4

Evaluate.

Implement	Explain
$\frac{3}{4} \cdot 2\frac{2}{9}$	Write the mixed number as an improper fraction.
$\frac{3}{4} \cdot \frac{20}{9}$	Simplify the fractions.
$\frac{3}{4} \cdot \frac{20}{9}$	Notice that $20 \div 4 = 5$ and $9 \div 3 = 3$
$\frac{1}{1} \cdot \frac{5}{3} = \frac{5}{3}$	You could also multiply straight across to get the answer $\frac{60}{36}$, then simplify the fraction to $\frac{5}{3}$. However, the method shown is more efficient.

Practice

Complete the problems on a separate sheet of paper.

Evaluate. Leave your final answers as improper fractions.

1) $-4\frac{2}{3} - \left(-\frac{4}{5}\right)$

2) $3\frac{3}{4} \cdot 2\frac{2}{13}$

3) $\frac{7}{12} - \frac{1}{3}$

4) $\frac{9}{10} \div 1\frac{1}{2}$

Practice Solutions

1) $-\frac{14}{3}+\frac{4}{5}$

$-\frac{70}{15}+\frac{12}{15}$

$-\frac{58}{15}$

2) $\frac{15}{4}\cdot\frac{28}{13}$

$\frac{15}{1}\cdot\frac{7}{13}$

$\frac{105}{13}$

3) $\frac{7}{12}-\frac{4}{12}$

$\frac{3}{12}=\frac{1}{4}$

The LCD of 12 and 3 is 12.

4) $\frac{9}{10}\div\frac{3}{2}$

$\frac{9}{10}\cdot\frac{2}{3}$

$\frac{3}{5}\cdot\frac{1}{1}$

$\frac{3}{5}$

Order of Operations

Prerequisite Skills: 6, 7, 11, 15

- Prior Math-U-See level
  - Pre-Algebra
    - Single Variable Equations (Lesson 9)
    - Order of Operations (Lesson 14)
- Additional Practice
  - Math-U-See Worksheet Generator located in the Digital Toolbox

- Objectives/Skills
  - Simplify using order of operations, including expressions containing absolute value and exponents.
  - Simplify an expression with terms to the second or third power (e.g., 3^2, 3^3).
  - Use the solution to an equation to evaluate an expression.
  - Use substitution. This includes checking a solution to a single variable equation.

Simplifying Expressions Using Order of Operations

When using the order of operations to simplify, keep the following in mind:

- Use the acronym PEMDAS to help you remember the order of operations
  - PEMDAS stands for Parentheses, Exponents, Multiplication/Division, and Addition/Subtraction.
- Parentheses means any grouping symbol including absolute value bars.
- Exponents include any square roots since exponents and square roots are related.
- Multiplication and division are completed at the same time going from the left of the expression to the right.
- Addition and subtraction are also completed at the same time going from left to right.

Example 1

Simplify.

Implement	**Explain**
$3^3 + 4 \div 2^2 - 4(1-3)^2 + \lvert -5+3 \rvert + \sqrt{25}$	Use order of operations (PEMDAS) as you simplify.
$3^3 + 4 \div \left(\frac{1}{2}\right)^2 - 4(-2)^2 + \lvert -2 \rvert + \sqrt{25}$	P: parentheses Simplify any grouping symbol including absolute value.
$3^3 + 4 \div \left(\frac{1}{2}\right)^2 - 4(-2)^2 + 2 + \sqrt{25}$	
$27 + 4 \div \frac{1}{4} - 4(4) + 2 + 5$	E: Exponents Simplify any exponents, including square roots.
$27 + 16 - 16 + 2 + 5$	MD: Multiply and Divide Multiply and divide from left to right. Remember when dividing fractions to multiply by the reciprocal: $4 \div \frac{1}{4} = 4 \cdot 4 = 16$
34	AS: Add and Subtract Add and subtract the remaining values from left to right.

Example 2

Simplify.

Implement	**Explain**
$-4\lvert -3-2\rvert + 2^3 - (5+1)^2 \div 12$	◀ P: grouping symbols including absolute value.
$-4\lvert -5\rvert + 2^3 - (6)^2 \div 12$	
$-4(5) + 2^3 - (6)^2 \div 12$	
$-4(5) + 8 - 36 \div 12$	◀ E: Exponents
$-20 + 8 - 3$	◀ MD: Multiply and divide from left to right.
-15	◀ AS: Add and subtract from left to right.

Evaluating Expressions Using Order of Operations

- Order of operations is also used when evaluating expressions.
    - In math, evaluating means to calculate the value of something.
- Evaluating is often used when an expression contains a variable.
    - If you know the value of the variable, you can evaluate the expression by substituting that value into the expression wherever that variable is.
    - Then, you can use order of operations to simplify the expression and determine its final value.

Example 1

Evaluate the expression.

Implement	**Explain**
$-a^2 + \frac{bc}{d}$ for $a = 1$, $b = 2$, $c = 3$, $d = 4$	◀ Substitute the given value of each variable into the expression.
$-(1)^2 + \frac{2(3)}{4}$	◀ Use the order of operations to simplify. P: None E: exponents MD
$-1 + \frac{6}{4}$	◀ Only 1 is being squared since the negative sign is outside of the parentheses. The negative is part of multiplication.
$-1 + \frac{3}{2} = -\frac{2}{2} + \frac{3}{2} = \frac{1}{2}$	◀ Simplify the fractions using the LCD.

Example 2

Evaluate the expression.

Implement	Explain
$xyz - x + y \div z^2$ for $x = 3, y = 12, z = -2$	Substitute the value of each variable into the expression.
$(3)(12)(-2) - 3 + 12 \div (-2)^2$	Order of operations P: None E: exponents
$(3)(12)(-2) - 3 + 12 \div 4$	MD: Multiply and divide from left to right.
$-72 - 3 + 3$	AS: Add and subtract from left to right.
-72	

Practice

Complete the problems on a separate sheet of paper.

Simplify.

1) $4^2 \div 8 + 3(1-2)^4 - |-5|$

2) $|3^3 - 2^5| + 15 \div 5 - 2 + \sqrt{16}$

3) $|5-(-2)| + 3|-4| \div 6 + 6(3)$

4) $6^2 \div 3 + 2^2(3-1)^2$

Evaluate.

5) $2xy^2 + 3x^3y$ for $x = -1, y = -2$

6) $-ab + \frac{b}{a}$ for $a = 5, b = 15$

7) $4x^2(3xy)^2$ for $x = 2, y = -3$

8) $ab + bc$ for $a = 4, b = 5, c = -1$

Practice Solutions

1) $4^2 \div 8 + 3(-1)^4 - 5$

$16 \div 8 + 3(1) - 5$

$2 + 3 - 5$

0

2) $|27 - 32| + 15 \div 5 - 2 + 4$

$|-5| + 15 \div 5 - 2 + 4$

$5 + 15 \div 5 - 2 + 4$

$5 + 3 - 4$

4

3) $|5 + 2| + 3(4) \div 6 + 6(3)$

$7 + 12 \div 6 + 18$

$7 + 2 + 18$

27

4) $6^2 \div 3 + 2^2(2)^2$

$36 \div 3 + 4(4)$

$12 + 16$

28

5) $2(-1)(-2)^2 + 3(-1)^3(-2)$

$2(-1)(4) + 3(-1)(-2)$

$-8 + 6$

-2

6) $-(5)(15) + \frac{15}{5}$

$-75 + 3$

-72

7) $4(2)^2(3 \cdot 2 \cdot -3)^2$

$4(4)(-18)^2$

$4(4)(324)$

$5{,}184$

8) $4(5) + 5(-1)$

$20 - 5$

15

Square Roots

Prerequisite Skill: 8

- Prior Math-U-See levels
  - Pre-Algebra (Lessons 8, 30)
- Additional Practice
  - Math-U-See Worksheet Generator located in the Digital Toolbox
- Objectives/Skills
  - Simplify square roots containing perfect square numbers.

Simplifying Square Roots

- Finding the square root of a value is the opposite of squaring.
  - $4 \cdot 4 = 16$
  - $\sqrt{16} = 4$
- A perfect square is any rational number multiplied by itself
  - A perfect square can be a fraction, but for now you will focus on the natural, or counting numbers ($1, 2, 3, 4, 5,$ etc.).
  - To find the square root of a perfect square, you find the value that when multiplied by itself results in the given product.
- To find the square root of a variable squared, the same method is used.
  - $x \cdot x = x^2$
  - $\sqrt{x^2} = x$
  - For variables, this is only true if the variable is positive, so pay close attention to the information given.

Example 1

Simplify. Assume All variables are positive.

Implement	**Explain**
A) $\sqrt{25} = \sqrt{5^2} = 5$	The square root of 25 is 5 because $25 = 5^2$.
B) $\sqrt{a^2} = a$	The square root of a^2 is a.
C) $-\sqrt{81} = -9$	Using order of operations, take the square root of 81 first. The square root of 81 is 9. Then, multiply 9 by the negative to get -9.
D) $\sqrt{10} = \sqrt{10}$	Not all numbers are perfect squares. Though you may be able to simplify a square root further (this will be covered in Algebra 1), it will not result in a rational number. For now, focus on the difference between a perfect square and a non perfect square.

Example 2

Identify the perfect squares from the given values.

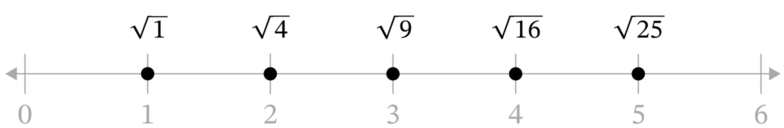
25, 32, 36, 47, 49, 72, 81

Implement

The perfect squares are 25, 36, 49, and 81.

Explain

$25 = 5^2$
$36 = 6^2$
$49 = 7^2$
$81 = 9^2$



This graph can help you correlate square roots and their equivalent natural number.

Practice

Complete the problems on a separate sheet of paper.

Simplify. Assume all variables are positive.

1) $\sqrt{81}$

2) $-\sqrt{1}$

3) $\sqrt{x^2}$

4) Name the perfect squares: $\sqrt{4}, \sqrt{5}, \sqrt{9}, \sqrt{13}$

Practice Solutions

1) 9

2) -1

3) x

4) $\sqrt{4}$, $\sqrt{9}$ are perfect squares

Equations and Solutions with Fractions

Prerequisite Skills: 3, 4, 5, 9, 10, 15

- Prior Math-U-See levels
    - Gamma (Lesson 8)
    - Epsilon
        - Simplifying Fractions (Lessons 12, 13)
        - Recording Mixed Numbers as Improper Fractions (Lesson 15)
        - Adding Fractions (Lessons 5, 8)
        - Subtracting Fractions (Lesson 5)
        - Multiplying Fractions (Lesson 9)
        - Dividing Fractions (Lesson 10)
        - Mixed Numbers (Lessons 17–25)
        - One and Two Step Equations with Variables and Fractions (Lesson 26)
    - Zeta (Lesson 19)
    - Pre-Algebra
        - Fractional Exponents (Lesson 5)
        - One and Two Step Equations with Variables and Integers (Lessons 9, 11, 13, 14)
- Objectives/Skills
    - Solve one- and two-step equations containing variables and integer coefficients.
    - Solve one- and two-step equations containing variables and fractions (coefficients and/or constant terms).
    - Use substitution. This includes checking a solution to a single variable equation.
- Additional Practice
    - Math-U-See Worksheet Generator located in the Digital Toolbox

Solving Equations Containing Fractions

- When solving equations, the goal is to isolate x (or other variable) on one side of the equation with the number on the other.
    - Remember, whatever you do to one side of an equation, you must do on the other side to maintain equality.
    - If a value is being added to x, use the additive inverse to make it equal to zero.

$$\begin{aligned} x - 2 &= 5 \\ +2 \ \ &\ \ +2 \\ x \qquad &= 7 \end{aligned}$$

    - If x is being multiplied by a value, use the multiplicative inverse to make it equal to one.

$$\begin{aligned} 3x &= 12 \\ \left(\tfrac{1}{3}\right)(3x) &= (12)\left(\tfrac{1}{3}\right) \\ x &= 4 \end{aligned}$$

    - Remember to combine any like terms before solving, including distributing fractions where necessary.
- When an equation contains fractions, it is still solved by isolating the variable.
    - However, there may be multiple ways of achieving this.
    - For now, continue to use the additive and multipicative inverses as you normally would and simplify the fractions as needed.
    - In Algebra 1, you will learn how to eliminate fractions or decimals before solving.

- When solving an equation from a word problem, read the problem carefully to make sure you understand what is happening.
    - Try to draw a sketch of the scenario if needed.
    - Look for key words that indicate any operations.

Example 1

Solve. Write the solution in simplest form.

$\frac{3}{5}x = 9$

Implement	Explain
$\frac{3}{5}x = 9$	Isolate x. Multiply each side of the equation by $\frac{5}{3}$, which is the reciprocal of $\frac{3}{5}$
$\left(\frac{5}{3}\right)\left(\frac{3}{5}x\right) = (9)\left(\frac{5}{3}\right)$	
$x = \frac{45}{3} = 15$	Multiply, then simplify the fraction.

Example 2

Solve. Write the solution in simplest form.

$\frac{2}{7}x + 6 = 2$

Implement	Explain
$\frac{2}{7}x + 6 = 2$	Isolate x. First, subtract 6 from both sides of the equation.
$-6 \quad -6$	
$\frac{2}{7}x = -4$	Then, multiply by the reciprocal of $\frac{2}{7}$.
$\left(\frac{7}{2}\right)\left(\frac{2}{7}x\right) = (-4)\left(\frac{7}{2}\right)$	
$x = \frac{-28}{2} = -14$	Multiply and then simplify.

Example 3

Solve. Write the solution in simplest form.

$-2\frac{1}{3}x + 5\frac{1}{4} = \frac{3}{4}$

Implement	Explain
$-2\frac{1}{3}x + 5\frac{1}{4} = \frac{3}{4}$	◀ First, change all mixed numbers to improper fractions.
$-\frac{7}{3}x + \frac{21}{4} = \frac{3}{4}$	◀ Isolate x. Subtract $\frac{21}{4}$ from both sides.
$\begin{array}{r} -\frac{7}{3}x + \frac{21}{4} = \frac{3}{4} \\ -\frac{21}{4} \quad -\frac{21}{4} \end{array}$	
$-\frac{7}{3}x = \frac{18}{4} \Rightarrow -\frac{7}{3}x = \frac{9}{2}$	◀ Simplify $\frac{18}{4}$. Multiply both sides by the reciprocal of $-\frac{7}{3}$.
$\left(-\frac{3}{7}\right)\left(-\frac{7}{3}x\right) = \left(\frac{9}{2}\right)\left(-\frac{7}{3}\right)$	
$x = -\frac{63}{6} = -\frac{21}{2}$	◀ Multiply and then simplify.

Example 4

Solve. Write the solution in simplest form.

$\frac{8}{5}(x + 10) = -16$

Implement	Explain
$\frac{8}{5}(x + 10) = -16$	◀ Distribute $\frac{8}{5}$.
$\frac{8}{5}x + 16 = -16$	◀ Subtract 16 from both sides of the equation.
$\begin{array}{r} \frac{8}{5}x + 16 = -16 \\ -16 \quad -16 \end{array}$	
$\frac{8}{5}x = -32$	◀ Multiply both sides by the reciprocal of $\frac{8}{5}$.
$\left(\frac{5}{8}\right)\left(\frac{8}{5}x\right) = (-32)\left(\frac{5}{8}\right)$	
$x = -20$	◀ Multiply and simplify: $-32 \div 8 = -4$ $-4 \cdot 5 = -20$

Example 5

Solve. Write the solution in simplest form.

$3x - 4 = 7x + 9$

Implement

$$3x - 4 = 7x + 9$$
$$-3x \qquad -3x$$

$$-4 = 4x + 9$$

$$-4 = 4x + 9$$
$$-9 \qquad -9$$

$$-\frac{14}{3} = \frac{3x}{3}$$

$$-\frac{14}{3} = x$$

Explain

- Subtract $3x$ from both sides of the equation.
- Subtract 9 from both sides of the equation.
- Divide both sides of the equation by 3 (this is the same as multiplying both sides of the equation by $\frac{1}{3}$).

Example 6

Solve. Write the solution in simplest form.

Susan bought packages of granola bars for a conference. After the conference, she noticed that $\frac{2}{3}$ were eaten. There are now 10 packages left. How many packages did she originally buy?

Implement

$$\frac{1}{3}p = 10$$

$$(3)\left(\frac{1}{3}p\right) = (10)(3)$$

$$p = 30$$

Explain

- Let $p =$ the number of packages that Susan bought.
  Since $\frac{2}{3}$ were eaten, this means $\frac{1}{3}$ of the packages are left.
  "One third of the packages were left." The word "of" means to multiply. "Were" is a "to be" verb which means equals.
  The number left is 10.
  Putting this together, you can say:
  "One third" times the packages equals 10." or $\frac{1}{3}p = 10$
- Multiply each side by the reciprocal of $\frac{1}{3}$, which is 3.

Susan bought 30 packages of granola bars.

Example 7

Solve. Write the solution in simplest form.

Daniel bought 4 shirts and a pair of pants. The pants were $25.50. He spent a total of $84.25. How much was each shirt?

Implement	Explain
$4x + 25.50 = 84.25$	Let $x =$ cost of one shirt. $4x =$ the total cost of 4 shirts The total cost of 4 shirts and the pair of pants: $4x + 25.50$ The total spent is 84.25. The four shirts plus the pair of pants equals 84.25: $4x + 25.50 = 84.25$
$4x + 25.50 = 84.25$ $-25.50 \quad -25.50$	Subtract 25.50 from both sides of the equation.
$\frac{4x}{4} = \frac{58.75}{4}$	Divide both sides of the equation by 4.
$x = 14.69$ (rounded) Each shirt cost $14.69	

Practice

Complete the problems on a separate sheet of paper.

Solve. Write the solution in simplest form.

1) $\frac{4}{5}x - 7 = 5$

2) $-\frac{1}{2}x + \frac{2}{3} = 1\frac{1}{3}$

3) $-3\frac{7}{8}x = 4\frac{1}{2}$

4) $\frac{5}{4}(x + 16) = 20$

5) $4x - 3 = 2x + 7$

6) Russell bought half of all of the available bunches of bananas. The store had 14 bunches left. How many bunches of bananas did the store originally have?

7) Donna bought 3 packages of strawberries and 2 packages of grapes. One package of grapes cost \$1.75. Donna spent a total of \$10.50. How much did one package of strawberries cost? (Round to the nearest cent.)

Practice Solutions

1)

$$\frac{4}{5}x - 7 = 5$$
$$+7 \quad +7$$
$$\frac{4}{5}x = 12$$
$$\left(\frac{5}{4}\right)\left(\frac{4}{5}x\right) = (12)\left(\frac{5}{4}\right)$$
$$x = 15$$

2)

$$-\frac{1}{2}x + \frac{2}{3} = \frac{4}{3}$$
$$-\frac{2}{3} - \frac{2}{3}$$
$$-\frac{1}{2}x = \frac{2}{3}$$
$$(-2)\left(-\frac{1}{2}x\right) = \left(\frac{2}{3}\right)(-2)$$
$$x = -\frac{4}{3}$$

3)

$$-\frac{31}{8}x = \frac{9}{2}$$
$$\left(-\frac{8}{31}\right)\left(-\frac{31}{8}x\right) = \left(\frac{9}{2}\right)\left(-\frac{8}{31}\right)$$
$$x = -\frac{36}{31}$$

4)

$$\frac{5}{4}x + 20 = 20$$
$$-20 \quad -20$$
$$\frac{5}{4}x = 0$$
$$\left(\frac{4}{5}\right)\left(\frac{5}{4}x\right) = (0)\left(\frac{4}{5}\right)$$
$$x = 0$$

5)

$$4x - 3 = 2x + 7$$
$$-2x \qquad -2x$$
$$2x - 3 = 7$$
$$+3 \quad +3$$
$$2x = 10$$
$$x = 5$$

6)

Let b = bunches of bananas

$$\frac{1}{2}b = 14$$
$$(2)\left(\frac{1}{2}b\right) = (14)(2)$$
$$b = 28$$

The store originally had
28 bunches of bananas.

7)

Let b = strawberries

$$3b + 2(1.75) = 14$$
$$3b + 3.50 = (14)(2)$$
$$3b + 3.50 = 28$$
$$-3.50 \qquad -3.50$$
$$3b = 7$$
$$b = \frac{7}{3} = 2\frac{1}{3}$$

Inequalities

Prerequisite Skills: 16, 17

- Prior Math-U-See levels
  - Beta
    - Inequalities (Lesson 3)
    - Number Lines (Appendix B)
  - Zeta
    - Number Lines (15G)

- Objectives/Skills
  - Solve one- and two-step inequalities, including inequalities with fractional coefficients.
  - Graph single variable inequalities on a number line.

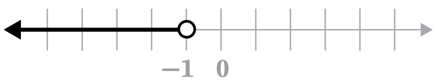
Operations with Fractions

- An inequality represents two values or expressions that are not equal to each other.
  - There are 4 symbols that represent inequalities:
    - ≤ less than or equal to
    - ≥ greater than or equal to
    - < less than
    - > greater than
- Inequalities are solved similarly to equations (isolating the variable) with a couple of differences:
  - If you multiply or divide an inequality by a negative number, you must flip the direction of the sign.
  - Inequalities will have infinite answers.
    - This means on a number line, the solution to an inequality will be a line of values with a single point representing the boundary of the inequality.
      - This point is either a closed point (filled in) or an open point (not filled in) depending on the symbol used.
        - Less than or equal to and greater than or equal to both have a closed point showing that the value on the number line is included as a possible solution.
        - Less than and greater than both have an open circle on the number line showing that the value is not included.

Example 1

Solve. Then graph the inequality on a number line.

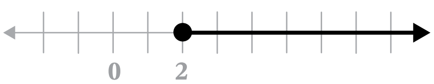
$2x - 7 < -9$

Implement	Explain
$2x - 7 < -9$ $+7 \quad +7$	Isolate x. Add 7 to both sides of the inequality.
$2x < -2$	Divide both sides of the inequality by 2.
$x < -1$	x is less than –1. This means x can be any value that is less than -1, but not including -1. So, x could be -5, -7.25, etc. continuing on infinitely to the left of -1 on the number line.
	To graph, put an open circle on -1 and shade all values on the number line that are less than –1. Make sure to put an arrow on the end of your line to show that it continues on infinitely in that direction.

Example 2

Solve. Then graph the inequality on a number line.

$x + 4 \geq 6$

Implement	Explain
$x + 4 \geq \ 6$ $\ \ -4 \quad -4$	Isolate x. Subtract 4 from both sides.
$x \geq 2$	
	The inequality is greater than or equal to 2, so put a closed circle on 2 since 2 is included as a possible value for x. Then, shade all values greater than 2 on the number line.

Example 3

Solve. Then graph the inequality on a number line.

$\frac{2}{3}(x - 3) > 4$

Implement	Explain
$\frac{2}{3}(x - 3) > 4$	Isolate x. Distribute $\frac{2}{3}$ across the parentheses.
$\frac{2}{3}x - 2 > 4$	Add 2 to both sides of the equation.
$\frac{2}{3}x > 6$	Multiply both sides of the equation by the reciprocal of $\frac{2}{3}$.
$\left(\frac{3}{2}\right)\left(\frac{2}{3}x\right) > (6)\left(\frac{3}{2}\right)$	You are not multiplying by a negative, so the inequality symbol does not change.
$x > 9$	Multiply 6 and $\frac{3}{2}$.
	x is greater than 9. Use an open circle on 9 for greater than and shade all numbers greater than 9 on the number line.

Practice

Complete the problems on a separate sheet of paper.

Solve. Then graph the inequality on a number line.

1) $x - 7 \geq -8$

2) $-3x + 1 \leq 6$

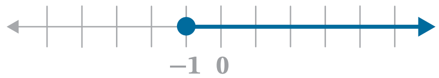
3) $\frac{2}{9}(x + 9) > 4$

Practice Solutions

1) $x - 7 \geq -8$

$\quad +7 \quad +7$

$x \geq -1$

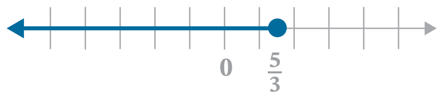


2) $3x + 1 \leq 6$

$\quad -1 \quad -1$

$3x \leq 5$

$x \leq \frac{5}{3}$



3) $\frac{2}{9}x + 2 > 4$

$\quad -2 \quad -2$

$\frac{2}{9}x > 2$

$\left(\frac{9}{2}\right)\left(\frac{2}{9}x\right) > (2)\left(\frac{9}{2}\right)$

$x > 9$

Geometry

Prerequisite Skills: 18 19, 20, 21, 22

- Prior Math-U-See levels
    - Beta
        - Formulas for Perimeter of a Triangle, Square, Rectangle, Trapezoid, and Circle (Lesson 15)
    - Gamma
        - Formula for Square and Rectangle (Lesson 7)
    - Delta
        - Formulas for Area of a Triangle (Lesson 9)
        - Formula for Volume of Prism (Lesson 26)
    - Zeta
        - Formula for Area and Circumference of a Circle (Lesson 16)
- Objectives/Skills
    - Reference a formula sheet to find an unknown formula.
    - Apply the following formulas for area: triangle, square, rectangle, trapezoid, and circle.
    - Apply the following formulas for perimeter: triangle, square, rectangle, trapezoid, and circle.
    - Apply the following formulas for volume: prism, cone, pyramid, sphere, and cylinder.
    - Label a solution with the correct units.
- Additional Practice
    - Math-U-See Worksheet Generator located in the Digital Toolbox

Using Geometric Formulas

- Before you can use a formula, you must first choose the correct formula for the problem you are solving.
    - Read the directions carefully and find the formula that contains both the value that you are solving for and the values that you already know.
- Once you have chosen a formula, substitute the values that you already know into it.
    - Then you can evaluate the expression remaining until it is fully simplified.
- Remember to always include the given units with your final answer. If no unit of measurement is specified, simply write “units” as the unit of measurement.
- The following formulas are used in the examples shown:
    - Perimeter
        - Rectangle: $2l + 2w = P$
        - Triangle: $a + b + c = P$
    - Volume
        - Cylinder: $V = r^2h$

Example 1

The perimeter of a rectangle is 48 units. The length is twice the width plus 3. What are the dimensions?

Implement	Explain
$P = 48$ $l = 2w + 3$	First, identify what you know from the problem: The perimeter of the rectangle is 48, or $P = 48$.
$2l + 2w = P$	The length (l) is two times the width (w) plus 3, or $l = 2w + 3$. The problem is asking for the dimensions (length and width) of the rectangle. This means the formula needed is $2l + 2w = P$.
$2(2w + 3) + 2w = 48$	Substitute $2w + 3$ for l and 48 for P. Then, solve for the value of w.
$4w + 6 + 2w = 48$ $6w + 6 = 48$	Distribute 2 and combine like terms.
$6w = 42$ $w = 7$	Subtract 6 from both sides of the equation, then divide by 6.
$l = 2(7) + 3$ $l = 17$	The width is 7. Now, substitute 7 into the equation $l = 2w + 3$ to find the length.
$2(17) + 2(7) = 48$ $34 + 14 = 48$	Check.
The dimensions of the rectangle are 17 units and 7 units.	Remember to include the appropriate units.

Example 2

The perimeter of a triangle is 30 inches. Side b is twice the length of side a, and side c is three times the length of side a. Find the dimensions of the triangle.

Implement	Explain
$P = 30$	Identify what you know: The perimeter is 30 inches, $P = 30$.
$b = 2a$	Side b is twice the length of side a, $b = 2a$.
$c = 3a$	Side c is three times the length of side a, $c = 3a$
$a + b + c = P$	The needed formula: $a + b + c = P$
$a + 2a + 3a = 30$	Substitute $b = 2a$ and $c = 3a$ into the formula and substitute 30 in for P.
$6a = 30$ $a = 5$	Combine like terms and solve for a.
$b = 2(5) = 10$ $c = 3(5) = 15$	Substitute 5 for a.
$5 + 10 + 15 = 30$	Check.
The dimensions of the triangle are 5 in, 10 in, and 15 in.	Remember to include the appropriate units.

Example 3

Find the height of a cylinder if the volume is 24π cm^3 and a radius of 4 cm.

Implement	Explain
$V = 24\pi \text{ cm}^3$ $r = 4$ $V = \pi r^2 h$	Identify what you know: The volume of the cube is 24π cm^3, or $V = 24\pi \text{ cm}^3$. The radius is 4 cm, or $r = 4$. The problem says to find the height. The formula needed: $V = \pi r^2 h$
$24\pi = \pi(4)^2 h$	Substitute all values.
$24\pi = 16\pi h$	Follow order of operations: $4^2 = 16$
$\frac{24\pi}{16\pi} = h$ $\frac{3}{2} = h$	Divide both sides of the equation by 16π. simplify to solve for h. Note that the value of π was not substituted in since it simplifies from the equation.

The height is $\frac{3}{2}$ cm.

Practice

Complete the problems on a separate sheet of paper.

Use the following formulas to find the missing information.

Rectangle
$\text{A} = lw$ or $\text{A} = bh$
$P = 2l + 2w$ or
$P = 2(l + w)$

Triangle
$\text{A} = \frac{1}{2}bh$ or $\text{A} = \frac{bh}{2}$
$P = a + b + c$

Rectangular Prism
$V = lwh$

Rectangular Pyramid
$V = \frac{1}{3}lwh$

1) The perimeter of a rectangle is 36 inches. The length is two more than the width. Find the dimensions of the rectangle.

2) The perimeter of a triangle is 19 cm. The largest side is twice the smallest side plus one. The middle side is two more than the smallest side. Find the dimensions of the triangle.

3) Find the height of a pyramid if the volume is 35 cm^3, the length of the base is 3 cm, and the width of the base is 5 cm.

4) Find the length of a rectangular prism if the height is 12 inches, the width is 4 inches, and the volume is 72 in^3.

Practice Solutions

1) $P = 2l + 2w; P = 36; l = 2w$

$36 = 2(2w) + 2w$

$36 = 4w + 2w$

$36 = 6w$

$w = 6$

$l = 12$

The dimensions of the rectangle are 6 in by 12 in.

2) $P = a + b + c; P = 12; c = 2a + 1; b = a + 2$

$19 = a + (a + 2) + (2a + 1)$

$19 = 4a + 3$

$16 = 4a$

$a = 4$

$b = 4 + 2 = 6$

$c = 2(4) + 1 = 9$

The dimensions of the triangle are 4 cm, 6 cm and 9 cm.

3) $V = \frac{1}{3}lwh; V = 35; l = 3, w = 5$

$35 = \frac{1}{3}(3)(5)h$

$35 = 5h$

$h = 7$

The height of the pyramid is 7 cm.

4) $V = lwh; V = 72; h = 12, w = 4$

$72 = l(4)(12)$

$72 = 48l$

$l = \frac{3}{2}$

The length is 1.5 in.

Number Relationships

Prerequisite Skills: 24, 25, 26, 27

- Prior Math-U-See levels
  - Gamma
    - Factors of a Number (Lesson 26)
  - Pre-Algebra
    - Proportions (Lessons 19, 20)
    - Least Common Multiple (Lesson 21)
    - Greatest Common Factor (Lesson 22)

- Objectives/Skills
  - Find the least common multiple (LCM) of a set of numbers.
  - Find the greatest common factor (GCF) of a set of numbers.
  - Name all factors of a number.
  - Use proportions to solve problems.

Number Relationships

- A factor is a number that multiplies with another number to form a product.
  - The greatest common factor (GCF) is the greatest factor that two or more numbers share.
  - To find the greatest common factor, list out the factors of all the numbers given and identify the greatest one that all the numbers have in common.
    - For some numbers, you may be able to determine the GCF in your head using mental math.
- A multiple is the product of a given number and another number.
  - The least common multiple (LCM) is the least number that is a multiple of two or more other numbers.
  - To find the least common multiple, list out the multiples of all the numbers given until you find the least one that all the numbers have in common.
- A proportion is two ratios that are equal to each other.
  - The proportion $\frac{2}{3} = \frac{8}{12}$ is read as “two is to eight as three is to twelve.”
  - Sometimes a proportion will be missing one of its values.
    - To find the missing value cross multiply the denominator of each ratio by the numerator of the other.
    - This will give you an equation that you can solve to find the missing value.

Example 1

Find the GCF and the LCM of 56 and 32.

Implement

GCF:

Factors of 56: 1, 2, 4, 7, <u>8</u>, 14, 28, 56

Factors of 32: 1, 2, 4, <u>8</u>, 16, 32

LCM:

56: 56, 112, 168, <u>224</u>, ...

32: 32, 64, 96, 128, 160, 192, <u>224</u>, ...

Explain

◀ Write out all factors of 56 and 32.

◀ The factor 8 is the greatest factor that 56 and 32 have in common.

◀ Write out the multiples of 32 and 56.

◀ The multiple 224 is the least common multiple that 56 and 32 have in common.

Example 2

Solve using cross multiplication.

$\frac{x}{3} = \frac{7}{15}$

Implement	Explain
$\frac{x}{3} = \frac{7}{15}$	◀ Cross multiply: Multiply 15 times x and 3 times 7.
$15x = 3(7)$	◀ Divide both sides of the equation by 15
$15x = 21$	
$x = \frac{21}{15} = \frac{7}{5}$	◀ Simplify the solution.

Practice

Complete the problems on a separate sheet of paper.

Determine the greatest common factor (GCF) or the least common multiple (LCM).

1) Find the GCF of 72 and 48

2) Find the LCM of 24 and 18

3) Find the GCF of 18 and 30

4) Find the LCM of 15 and 25

Solve the following proportion. Write the solution in simplest form.

5) $\frac{x}{7} = \frac{10}{21}$

Practice Solutions

1) Factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, <u>24</u>, 72
Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, <u>24</u>, 48

The GCF of 72 and 48 is 24.

2) Multiples of 24: 24, 48, <u>72</u>
Multiples of 18: 18, 36, 54, <u>72</u>

The LCM of 24 and 18 is 72

3) Factors of 18: 1, 2, 3, <u>6</u>, 9, 18
Factors of 30: 1, 2, 3,5, <u>6</u>,10, 15, 30

The GCF of 18 and 30 is 6

4) Multiples of 15: 15, 30, 45, 60, <u>75</u>
Multiples of 25: 25, 50, <u>75</u>

The LCM of 15 and 25 is 75.

5) $21x = 70$

$x = \frac{70}{21}$

$x = \frac{10}{3}$

Math Vocabulary

Prerequisite Skills: 12, 13, 23

- Prior Math-U-See levels
    - Alpha
        - Know Vocabulary Sum (Lesson 7)
        - Difference (Lesson 18)
    - Gamma
        - Product (Lesson 1)
    - Delta
        - Quotient (Lesson 4)
    - Zeta
        - One Variable Equation from a Word Problem (Lesson 19)

- Objectives/Skills
    - Write a one-variable equation from a word problem and solve it.
    - Create an equation from a word problem.
    - Use vocabulary such as sum, difference, quotient, product, and other words that determine operations.

Using Math Vocabulary

- When solving word problems, there are some key vocabulary words that can help you determine what operations to use:
    - Multiply: of, product, per (depending on context), twice, times
    - Divide: per (depending on context), quotient, divided by
    - Add: and, sum, plus, add, more than, increase
    - Subtract: difference, minus, subtract, less, less than, decreased
    - Equality: is, the same as, equal to
    - When a problem uses the phrase "a number," this represents an unknown value, n.
- Always read problems carefully to make sure you understand what is happening. Without knowing the context, you may use the wrong operations for certain key terms.

Example 1

Write an equation for the given information.

Implement		Explain
A)	The difference of a number and 7 $n - 7$	“Difference” represents subtraction. “A number” represents an unknown value, n.
B)	The product of a number and 5 $5n$	“Product” represents multiplication. “A number” represents an unknown value, n.
C)	The sum of a number and 8 $n + 8$	“Sum” represents addition. “A number” represents an unknown value, n.
D)	The quotient of a number and 10 $\frac{n}{10}$	“Quotient” represents division. “A number” represents an unknown value, n.
E)	34 less than a number $n - 34$	“Less” represents subtraction. “A number” represents an unknown value, n.
F)	Four times the sum of a number and 3 $4(n + 3)$	“Times” represents multiplication. “Sum” represents addition. “A number” represents an unknown value, n.

Example 2

Write an equation for the given information and solve.

The quotient of a number and five, plus 2 is the same as ten.

Implement	Explain
The quotient of a number and five, plus 2 is the same as ten.	“Quotient” represents division. “And” represents addition. “A number” represents an unknown value, n. “The same as” represents equality or equal to.
$\frac{n}{5} + 2 = 10$	Subtract two from both sides of the equation
$\frac{n}{5} = 8$	Multiply both sides by 5
$n = 40$	

Example 3

Write an equation for the given information and solve.

Tate is paid $12 per hour and has already saved $120. How many hours does he need to work to have $360?

Implement	Explain
Tate is paid $12 per hour and has already saved $120. How many hours does he need to work to have $360?	◀ Per in this context represents multiplication, $12 per hour, or $12h$. Since he has already earned $120, add that to $12h$. The equation should equal $360.
$12h + 120 = 360$	◀ Subtract 120 from both sides of the equation
$12h = 240$	◀ Divide both sides by 12
$h = 20$	◀ Tate needs to work 20 hours to have $360.

Practice

Complete the problems on a separate sheet of paper.

Write an equation for the given information, and then solve.

1) The quotient of a number and 4 is 10

2) The product of -7 and a number, plus 12 is the same as -9.

3) Three times the sum of a number and 5 is 15.

4) Nine less than a number is two.

5) Sara earns $\$50$ for every lawn she mows. She has already mowed 3 lawns. She wants to save $\$750$ this summer. How many more lawns does she need to mow?

Practice Solutions

1) $\frac{n}{4} = 10$

$n = 40$

2) $-7n + 12 = -9$

$-7n = -21$

$n = 3$

3) $3(n + 5) = 15$

$3n + 15 = 15$

$3n = 0$

$n = 0$

4) $n - 9 = 2$

$n = 11$

5) Let $l =$ the number of lawns mowed.

$50l + 3(50) = 750$

$50l + 150 = 750$

$50l = 600$

$l = 12$

Sara needs to mow 12 lawns to have $750.

Coordinate Plane

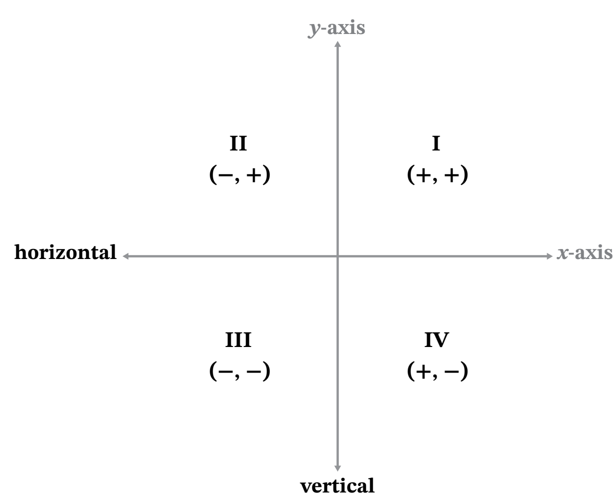
Prerequisite Skills: 28, 29, 30

- Prior Math-U-See levels
  - Zeta
    - Positive Coordinates (16G)
    - Plotting Coordinates (17G)
    - Distance with Coordinates (21G)

- Objectives/Skills
  - Name each quadrant and axis of the coordinate plane.
  - Plot ordered pairs, (x, y), in any quadrant of the coordinate plane.
  - Determine the horizontal or vertical distance between ordered pairs (points) on the coordinate plane.

The Coordinate Plane

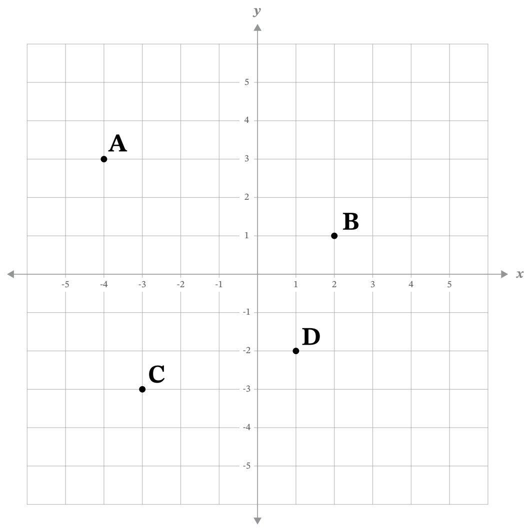
- The coordinate plane is made up of a horizontal number line and a vertical number line.
  - The horizontal number line is called the x-axis and represents the coordinates of x.
  - The vertical number line is called the y-axis and represents the coordinates for y.
- When x and y coordinates are combined, they form a single point on the coordinate plane.
  - Points are always written as (x, y). In other words, the x-coordinate is written first, followed by the y-coordinate.
  - To plot a point on the coordinate plane, find the place where the x-value and y-value meet then mark a dot at that point.
- The point where the x- and y-axes cross is called the origin and is represented by the point $(0, 0)$ because it is the point where both x and y are equal to zero.
- The coordinate plane is separated into four sections called quadrants.
  - These quadrants are labeled going counter-clockwise around the plane.
  - For all coordinates (x, y):
    - Quadrant I: x and y are positive
    - Quadrant II: x is negative and y is positive
    - Quadrant III: x and y are both negative
    - Quadrant IV: x is positive and y is negative



Example 1

Plot the given points.

A:(−4, 3) B: (2,1) C: (−3,−3) D: (1, −2)

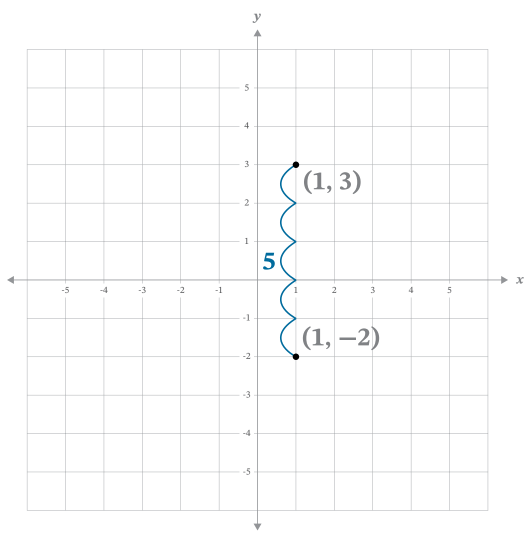


◀ Find the point where each x and y value meets. Mark it with a dot and then write the corresponding letter next to the point.

Example 2

Find the distance between the given points.

Find the distance between (1, −2) and (1, 3)



◀ Plot the points on the coordinate plane.

◀ The points have the same x value, so you only need to find the vertical distance.

This is the distance between the two y values.

◀ Count the vertical distance from −2 to 3.

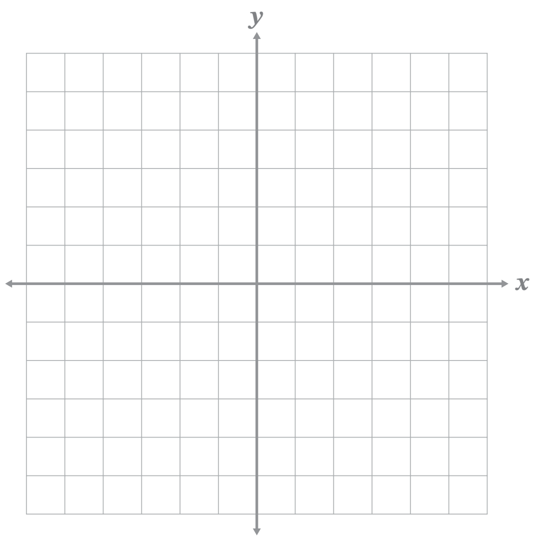
◀ The distance is 5.

Practice

Complete the problems on a separate sheet of paper.

Plot each point. Then name the quadrant the point is located in.

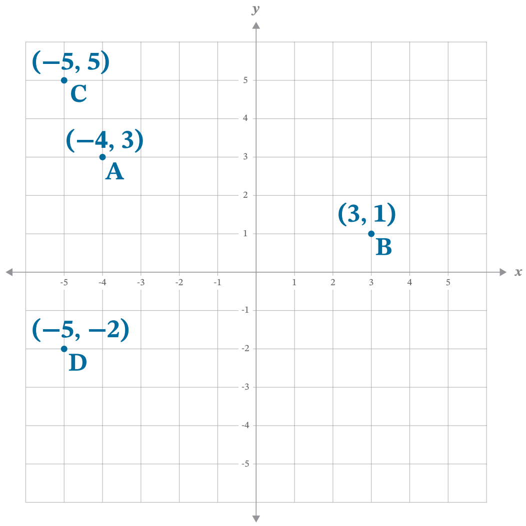
1) A: $(-4, 3)$
B: $(3, 1)$
C: $(-5, 5)$
D: $(-5, -2)$



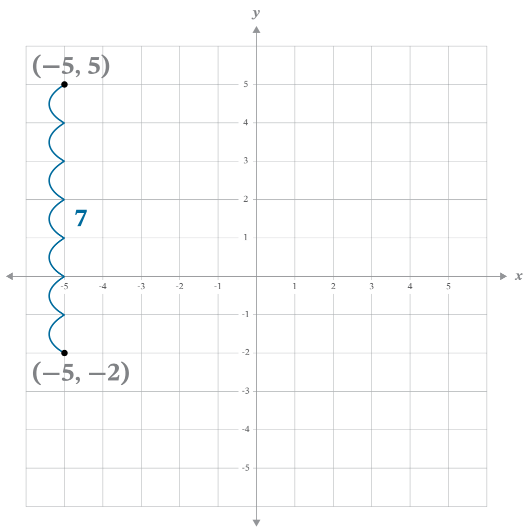
2) Find the distance between points C and D from problem 1.

Practice Solutions

1) A: QII
B: QI
C: QII
D: QIII



2) The distance is 7.



Complete a Table of Values

Prerequisite Skills: 31, 32

- Prior Math-U-See levels
    - Zeta
        - Plotting Coordinates (17G)
        - Tables of Values (18G)
        - Distance with Coordinates (21G)

- Objectives/Skills
    - Given an equation with two variables, use the substitution property to find a missing variable.
    - Complete a table by solving for the missing values.

Completing a Table of Values

- A table of values represents the corresponding values for two categories or variables.
- When given an equation containing two variables, you can use the equation to complete the table.
    - To find the missing values, subsitute the values for the known variable one at a time into the equation. Then, solve the equation for the missing value.

Example 1

Complete the table of values for the given equation.

$y = 12x - 3$

x	y
-2	
0	
2	

Implement

$y = \frac{1}{2}(-2) - 3 = -1 - 3 = -4$

$y = \frac{1}{2}(0) - 3 = 0 - 3 = -3$

$y = \frac{1}{2}(2) - 3 = 1 - 3 = -2$

x	y
-2	-4
0	-3
2	-2

Explain

◀ Substitute each value of x into the equation to find the value of y.

◀ Fill in the table with the corresponding y values.

Practice

Complete the problems on a separate sheet of paper.

Complete the table using the given values and equations.

1) $y = 3x + 2$

x	y
-2	
-1	
0	
1	

2) $y = -5x$

x	y
-1	
0	
1	
2	

Practice Solutions

1)

x	work	y
-2	$y = 3(-2) + 2 = -6 + 2$	-4
-1	$y = 3(-1) + 2 = -3 + 2$	-1
0	$y = 3(0) + 2 = 0 + 2$	2
1	$y = 3(1) + 2 = 3 + 2$	5

2)

x	work	y
-1	$y = -5(-1)$	5
0	$y = -5(0)$	0
1	$y = -5(1)$	-5
2	$y = -5(2)$	-10