Lesson 19
Product Rules for Exponents

Outline

Part A
Product and Power Rules for Exponents

- Rule 1: Product of Powers
- Rules 2 and 3: Power Rules for Exponents

Part B
Applications of Exponent Rules 1–3

- More Than One Exponent Rule
- Formulas with Exponents

Targeted Review

Vocabulary

- power

Warm Up

Q: What is another name for an exponent?

A: *Power*

Q: Does the diagram represent an expression or equation? Explain.

A: *This is an expression because there is no equals sign.*

19A EXPLORE

Part A: Product and Power Rules for Exponents

Objectives

In this part of the lesson, you will learn about product and power rules for exponents.

By the end of this lesson, you will be able to do the following:

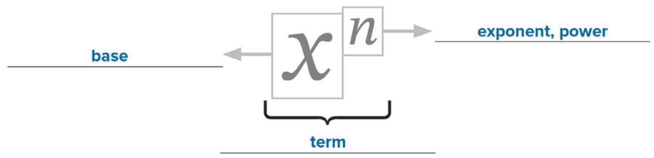
- Use the product of powers rule for exponents $(a^x \cdot a^y = a^{x+y})$ to simplify expressions.
- Use the power of powers rule for exponents $\left((a^x)^y = a^{x \cdot y}\right)$ and the power of a product rule for exponents $\left((ab)^x = a^x b^x\right)$ to simplify expressions.

Why?

The product and power rules for exponents allow you to multiply and factor terms that are raised to a power. You will use these rules throughout this unit and in future lessons.

Warm Up

Label the diagram using exponent, base, power, and term.



Rule 1: Product of Powers

- Adding like terms changes the **coefficient**, but *multiplying* like terms changes the **power of the base**:
  - $x + x =$ **$2x$**
  - $x \cdot x =$ **x^2**
- The **power** (or exponent) represents the number of times that you multiply the base by itself.
- When the power is not visible, its value is **one**.

EXPLORE 19A

- The rules for exponents are used when you are **multiplying** bases together.
- Rule 1 is the product of powers rule for exponents:
  - When **like** bases are multiplied together, the exponents of those bases are **added**.
  - Rule 1 product of powers: For all real numbers, $a^x \cdot a^y = a^{x+y}$.

Example 1

Expand the exponential expression. Then simplify the expression to one term using the product of powers rule.

A) $x^5 \cdot x^3$

Expanded: $(x \cdot x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x)$

Simplified using the product of powers rule: $x^5 \cdot x^3 = x^{(5+3)} = x^8$

B) $a^4b^2 \cdot a^3b$

Expanded: $(a \cdot a \cdot a \cdot a \cdot b \cdot b) \cdot (a \cdot a \cdot a \cdot b)$

Simplified using the product of powers rule:

$a^4b^2 \cdot a^3b$

$a^{4+3} \cdot b^{2+1}$

a^7b^3

Example 2

Expand the exponential expression. Then simplify the expression to one term using the product of powers rule.

A) $5^4 \cdot 5^2$

Expanded: $(5 \cdot 5 \cdot 5 \cdot 5) \cdot (5 \cdot 5)$

Exponent Rule: $5^4 \cdot 5^2 = 5^{4+2} = 5^6$

B) $2^3 3^4 \cdot 2^2 3^2$

Expanded: $(2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3) \cdot (2 \cdot 2 \cdot 3 \cdot 3)$

Use the Commutative Property to reorder the terms: $(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$

Exponent Rule: $2^3 3^4 \cdot 2^2 3^2 = 2^{3+2} \cdot 3^{4+2} = 2^5 3^6$

> Our focus is on the exponents, but if you want to know this numerical value, you first need to find the value of each base.
> $2^5 = 32$
> $3^6 = 729$
> $32 \cdot 729 = 23{,}328$

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: Why is it not possible to combine all of the exponents in this expression?

A: *Because you can only add exponents of like-bases.*

Q: Why is using the exponent rules more efficient than expanding to find the value?

A: *Sample: Expanding the expressions takes a long time, and it is easy to make a mistake if the exponent is a large number.*

Example 3

Find the missing number using the product of powers rule.

Solve for the value of the missing exponent. The simplified answer is given, but the exponent of one of the bases is missing. As long as *all of the bases are the same*, you can remove the bases and compare the exponents.

A) Find the value of the missing exponent.

$x^3 \cdot x^? = x^{11}$

$3 + ? = 11$

$? = 11 - 3$

$? = 8$

The exponent rules work for all real number exponents, including exponents that are negative numbers or fractions.

B) Find the value of n.

$2^n \cdot 2^6 \cdot 2^7 = 2^8$

$n + 6 + 7 = 8$

$n + 13 = 8$

$n = 8 - 13$

$n = -5$

Checkpoint

Expand the expression. Then simplify using the product of powers rule for exponents.

$2^3a^2 \cdot 2^8a^5$

Expanded: $(2 \cdot 2 \cdot 2 \cdot a \cdot a) \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot a \cdot a \cdot a \cdot a \cdot a)$

Exponent Rule: $2^3a^2 \cdot 2^8a^5 = 2^{3+8}a^{2+5}$

$2^{11}a^7$

Rules 2 and 3: Power Rules for Exponents

- Rule 2 is the power of a **power** rule for exponents:
  - When a power is raised to a power, the exponents are **multiplied**.
  - For all real numbers, $(a^x)^y = a^{x \cdot y}$.
- Rule 3 is the power of a **product** rule for exponents:
  - If more than one base within parentheses is raised to a power, the exponent is **distributed** to *each* base.
  - For all real numbers, $(ab)^x = a^x b^x$.
- Most often, a **combination** of Rules 2 and 3 is needed.
  - For all real numbers, $(a^x b^y)^z = a^{x \cdot z} b^{y \cdot z}$.

EXPLORE 19A

Example 4

Expand the exponential expression using the outermost exponent. Then simplify the expression using the power of a power rule.

A) $(x^{30})^2$

Expanded: $x^{30} \cdot x^{30} = x^{30+30} = x^{60}$

Exponent rule: $x^{30 \cdot 2} = x^{60}$

B) $(x^{\frac{3}{2}})^3$

Expanded: $x^{\frac{3}{2}} \cdot x^{\frac{3}{2}} \cdot x^{\frac{3}{2}} = x^{\frac{3}{2}+\frac{3}{2}+\frac{3}{2}} = x^{\frac{9}{2}}$

Exponent rule: $x^{\frac{3}{2} \cdot 3} = x^{\frac{9}{2}}$

Example 5

Simplify the expression using exponent rules. Write any numerical coefficients without an exponent.

A) $(a^4b^7c)^3$

$a^{4 \cdot 3}b^{7 \cdot 3}c^{1 \cdot 3}$

$a^{12}b^{21}c^3$

B) $(2x^3y^4)^4$

$2^{1 \cdot 4}x^{3 \cdot 4}y^{4 \cdot 4}$

$2^4x^{12}y^{16}$

$16x^{12}y^{16}$

☑ Checkpoint

Explain the difference between Rule 1 and Rule 2.

With Rule 1, you add the exponents when multiplying bases that are the same.
With Rule 2, you multiply the exponents when a power is raised to another power.

Simplify.

A) $x^3 \cdot x^3$

x^{3+3}

x^6

B) $(x^3)^3$

$x^{3 \cdot 3}$

x^9

☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

These problems show how important it is to understand the difference between Rule 1 and Rule 2 for exponents.

Worked solutions for these problems are located in the Digital Pack.

For Practice 1 and Practice 2, there is a slight increase in the number of practice problems from previous lessons. This is to help students who have no prior experience with exponent rules. If your student has shown mastery throughout the lesson, it is not necessary that they complete every problem. Instead, you can assign them a few problems from each section to verify their mastery of the concept.

9)
Q: What property allows you to reorder the variables in your expression?
A: *The Commutative Property.*

12)
Q: Why should the coefficients be left in exponential form?
A: *Because the numbers are too large to find the value quickly.*

15)
Q: Which of the four operations did you use to simplify problems 3–15?
A: *Addition*

16–18)
Q: How is finding the value of n in these problems similar to solving equations?
A: *It is similar because you are comparing equal values to find an unknown value.*

29)
Q: If there are 3 bases in a term being raised to a power, which bases should the exponent be distributed to?
A: *It should be distributed to all of the bases.*

33)
Q: Which of the four operations did you use to simplify problems 19–33 when an exponent was raised to a power?
A: *Multiplication*

19A PRACTICE 1

Practice 1

Complete practice problems on a separate sheet of paper.

For problems 1–2, expand.

1) x^5 $\boldsymbol{x \cdot x \cdot x \cdot x \cdot x}$

2) 6^2x^4 $\boldsymbol{6 \cdot 6 \cdot x \cdot x \cdot x \cdot x}$

For problems 3–15, simplify. Assume all variables are positive.

3) $x^8 \cdot x^2$ $\boldsymbol{x^{10}}$

4) $2^8 \cdot 2^{11}$ $\boldsymbol{2^{19}}$

5) $3^5 \cdot 3^8 \cdot 3^{-3}$ $\boldsymbol{3^{10}}$

6) $x^2 \cdot x^2 \cdot x$ $\boldsymbol{x^5}$

7) $x^2y^2 \cdot xy^5$ $\boldsymbol{x^3y^7}$

8) $x^3y^9 \cdot x^2y$ $\boldsymbol{x^5y^{10}}$

9) $x^8y^{-2}z^{-3} \cdot xy^4z^8$ $\boldsymbol{x^9y^2z^5}$

10) $2^2y^3 \cdot 2^3y^8$ $\boldsymbol{2^5y^{11}}$

11) $5^8xy \cdot 5x^2y^4$ $\boldsymbol{5^9x^3y^5}$

12) $2^95^3 \cdot 2^65^{11}$ $\boldsymbol{2^{15}5^{14}}$

13) $(ab)(ab)$ $\boldsymbol{a^2b^2}$

14) $(3^7x^5)\cdot(5^5x^{12})$ $\boldsymbol{3^75^5x^{17}}$

15) $(a^2b^2c^2)(abc)$ $\boldsymbol{a^3b^3c^3}$

For problems 16–18, find the value of n.

16) $y^5 \cdot y^3 \cdot y^n = y^{15}$ $\boldsymbol{n = 7}$

17) $x^{-3} \cdot x^n \cdot x^5 = x^8$ $\boldsymbol{n = 6}$

18) $(11^n)^7 = 11^{35}$ $\boldsymbol{n = 5}$

For problems 19–30, simplify.

19) $(17^8)^{\frac{1}{2}}$ $\boldsymbol{17^4}$

20) $\left(x^{\frac{2}{3}}\right)^{18}$ $\boldsymbol{x^{12}}$

21) $\left(y^{\frac{11}{3}}\right)^{\frac{3}{2}}$ $\boldsymbol{y^{\frac{11}{2}}}$

22) $(a^3)^3$ $\boldsymbol{a^9}$

23) $(x^9)^5$ $\boldsymbol{x^{45}}$

24) $(y^3)^2$ $\boldsymbol{y^6}$

25) $(x^2y^3)^5$ $\boldsymbol{x^{10}y^{15}}$

26) $(xy^2)^3$ $\boldsymbol{x^3y^6}$

27) $(p^8q^{14})^{\frac{1}{2}}$ $\boldsymbol{p^4q^7}$

28) $(x^2y)^4$ $\boldsymbol{x^8y^4}$

29) $(ab^3c)^7$ $\boldsymbol{a^7b^{21}c^7}$

30) $(p^{27}q^{39})^{\frac{1}{3}}$ $\boldsymbol{p^9q^{13}}$

For problems 31–33, simplify. Write any numerical coefficients without an exponent.

31) $(2x)^2$ $\boldsymbol{4x^2}$

32) $(5y)^3$ $\boldsymbol{125y^3}$

33) $(3x^8)^3$ $\boldsymbol{27x^{24}}$

6 Lesson 19: Product Rules for Exponents > Part A: Product and Power Rules for Exponents > Practice 1 Algebra 1 Student Worktext

Mastery Check

Show What You Know

Complete the statement with one of the following words: always, sometimes, never.

If you use the word sometimes or never, provide a counterexample that shows why it is not always true.

A) The exponent rules are ___**always**___ true for all real numbers.

B) When using the product and power rules for exponents (Rules 1, 2, and 3), you will

___**sometimes**___ *add* the exponents.

Sample counterexamples:
When the bases are different, you cannot add the exponents (e.g., $2^3 \cdot x^8$ cannot be further combined).

When a power is raised to another power, the exponents are multiplied $\left(\text{e.g., } (x^5)^3 = x^{15}\right)$.

C) Simplify the expression using exponent rules. Write any numerical coefficients without an exponent.

$x^2 \cdot (3y^5)^2$

$\mathbf{x^2 \cdot 3^2 y^{10}}$

$\mathbf{9x^2y^{10}}$

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Show What You Know

A) Have your student review their guided notes if they are unsure whether the rules are true for all real numbers.

C)

Q: What rule or rules allow you to distribute the exponent across the parentheses?

A: *Rules 2 and 3.*

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Use the product of powers rule for exponents $\left(a^x \cdot a^y = a^{x+y}\right)$ to simplify expressions.
- Use the power of powers rule for exponents $\left((a^x)^y = a^{x \cdot y}\right)$ and the power of a product rule for exponents $\left((ab)^x = a^x b^x\right)$ to simplify expressions.

Worked solutions for these problems are located in the Digital Pack.

4)

Q: What should you do to simplify the bases? Can you multiply different numbers together that have different exponents?

A: *Simplify each base first, then multiply the numbers together. You cannot multiply different numbers with different exponents until they are both simplified to the first power.*

30)

Q: This is a hint to simplify the coefficient. What is the square root of 64?

A: *8*

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

Practice 2

Complete practice problems on a separate sheet of paper.

For problems 1–2, simplify. Assume all variables are positive.

1) $x^{15} \cdot x^{-12} \cdot x$ $\mathbf{x^4}$

2) $a^{\frac{2}{3}} \cdot a^{\frac{2}{3}} \cdot a^{\frac{5}{3}}$ $\mathbf{a^3}$

For problems 3–4, simplify. Write your final answer with no exponents.

3) $2^3 \cdot 3^2$ **72**

4) $5^2 \cdot 2^4$ **400**

For problems 5–8, simplify.

5) $a^5b^2 \cdot a^7b^8$ $\mathbf{a^{12}b^{10}}$

6) $9^9x^2y^3 \cdot (9^3x^2y)$ $\mathbf{9^{12}x^4y^4}$

7) $(m^3n^7) \cdot (m^{14}n^{-5})$ $\mathbf{m^{17}n^2}$

8) $(xyz) \cdot (x^2yz^3)$ $\mathbf{x^3y^2z^4}$

For problems 9–10, simplify. Assume all variables are positive.

9) $3x^2 \cdot 2x^3$ $\mathbf{6x^5}$

10) $8xy^2 \cdot 3xy^2$ $\mathbf{24x^2y^4}$

For problems 11–13, simplify.

11) $(12^5)^3$ $\mathbf{12^{15}}$

12) $(26^2)^4$ $\mathbf{26^8}$

13) $(5^7)^8$ $\mathbf{5^{56}}$

For problems 14–15, find the value of n.

14) $5^n \cdot 5^{-6} \cdot 5^3 = 5^1$ $\mathbf{n = 4}$

15) $(a^3)^n = a^{33}$ $\mathbf{n = 11}$

For problems 16–18, simplify. Write improper fractions where needed.

16) $(2^8)^{\frac{2}{3}}$ $\mathbf{2^{\frac{16}{3}}}$

17) $\left(x^{\frac{1}{2}}\right)^{12}$ $\mathbf{x^6}$

18) $\left(8^{\frac{1}{2}}\right)^2$ **8**

For problems 19–27, simplify.

19) $(12^5y)^2$ $\mathbf{12^{10}y^2}$

20) $(x^5y^8)^4$ $\mathbf{x^{20}y^{32}}$

21) $(7^4x)^5$ $\mathbf{7^{20}x^5}$

22) $(a^2b^3c^4)^2$ $\mathbf{a^4b^6c^8}$

23) $(m^2n^8)^9$ $\mathbf{m^{18}n^{72}}$

24) $(4xy^2)^4$ $\mathbf{4^4x^4y^8}$

25) $(a^{11}b^5)^8$ $\mathbf{a^{88}b^{40}}$

26) $(m^{20}n^6)^{\frac{1}{2}}$ $\mathbf{m^{10}n^3}$

27) $(x^9b^{16})^2$ $\mathbf{x^{18}b^{32}}$

For problems 28–30, simplify. Write the numbers as a single term.

28) $(2^2 \cdot 3)^2$ **144**

29) $(5x^2yz)^3$ $\mathbf{125x^6y^3z^3}$

30) $(8^2x^8y^{20})^{\frac{1}{2}}$ $\mathbf{8x^4y^{10}}$

EXPLORE 19B

Part B: Applications of Exponent Rules 1–3

Objectives

In this part of the lesson, you will learn about applications of exponent rules 1–3.

By the end of this lesson, you will be able to do the following:

- Simplify exponential expressions using both the product and power rules for exponents.
- Apply the product and power rules for exponents to formulas.

Why?

How can you determine the optimal size for a container? Being able to apply exponent rules to formulas will allow you to answer real-life questions like this.

Warm Up

Use the formula to find the area or volume of the figure.

1) Find the area of a rectangle with a length of 12 yards and a width of 5 yards.
$A = lw; l = 12, w = 5$
$A = (12)(5) = 60 \text{ yd}^2$

2) Find the area of a square with sides of 9 feet.
$A = s^2; s = 9$
$A = (9)^2 = 81 \text{ ft}^2$

3) Find the volume of a cube, $V = s^3$, when each edge has a length of 2 inches.
$V = s^3; s = 2$
$V = (2)^3 = 8 \text{ in}^3$

More Than One Exponent Rule

- Use the ___order of operations___ to determine which parts of an expression to simplify first.
- An expression is considered ___simplified___ when each base occurs only ___once___.

Warm Up

Remember to use your Formula Sheet.

Q: How will the units of a figure be labeled when finding the area?
A: *Square units*

Q: How will the units of a figure be labeled when finding the volume?
A: *Cubic units*

19B EXPLORE

Example 1

Simplify.

$\left(2x^{8}y^{6}\right)^{3} \cdot \frac{3}{5}x^{-3}$

Plan Determine the exponent rules needed to simplify.

Implement	Explain
$\left(2x^{8}y^{6}\right)^{3} \cdot \frac{3}{5}x^{-3}$	◂ Given
$2^{1 \cdot 3}x^{8 \cdot 3}y^{6 \cdot 3} \cdot \frac{3}{5} \cdot x^{-3}$	◂ Rules 2 and 3
$2^{3} \cdot \frac{3}{5} \cdot x^{24+(-3)}y^{18}$	◂ Commutative Property
$8 \cdot \frac{3}{5} = \frac{24}{5}$	◂ Simplify numerical bases
$\frac{24}{5}x^{21}y^{18}$	◂ Rule 1

Example 2

Simplify.

$\left(8m^{4}n^{3}\right)^{7} \cdot \left(3^{4}m\right)^{5}$

Plan Determine the exponent rules needed to simplify.

Implement	Explain
$\left(8m^{4}n^{3}\right)^{7} \cdot \left(3^{4}m\right)^{5}$	◂ Given
$8^{1 \cdot 7}m^{4 \cdot 7}n^{3 \cdot 7} \cdot 3^{4 \cdot 5}m^{1 \cdot 5}$	◂ Rules 2 and 3
$8^{7} \cdot 3^{20} \cdot m^{28+5}n^{21}$	◂ Commutative Property
$8^{7} \cdot 3^{20}m^{33}n^{21}$	◂ Rule 1

☑ Checkpoint

Simplify.

$\left(5a^{4}b^{3}\right)^{8} \cdot \left(5a^{6}\right)^{2}$

$5^{1 \cdot 8}a^{4 \cdot 8}b^{3 \cdot 8} \cdot 5^{1 \cdot 2}a^{6 \cdot 2} = 5^{8+2}a^{32+12}b^{24}$

$5^{10}a^{44}b^{24}$

☑ Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: What is the first step to simplifying the expression?

A: *Distribute the exponents across the bases in the parentheses.*

Q: Why should the coefficient be left in exponential form?

A: *Because the value is too large to calculate with mental math.*

EXPLORE 19B

Formulas with Exponents

- When working with formulas and terms that contain exponents, start with the general formula and then substitute in the known values.

Example 3

Solve for the area of a square with sides of $11p^4q$ units.

Plan	Implement	Explain
$A = s^2$	$A = (11p^4q)^2$	Substitution
$s = 11p^4q$	$A = 11^{1\cdot 2}p^{4\cdot 2}q^{1\cdot 2}$	Rules 2 and 3
Use substitution and solve for A.	$A = 121p^8q^2$ square units	

Remember to label area problems with square units of measurement. Label volume problems with cubic units.

Example 4

Find the volume of a sphere with a radius of $2x^3y^2$ centimeters.

Plan Write the formula for the volume of a sphere.
Replace "r" with the given value.

Implement	Explain
$V = \frac{4}{3}\pi r^3$	Volume of a sphere
$V = \frac{4}{3}\pi(2x^3y^2)^3$	Substitution
$V = \frac{4}{3}\pi \cdot 2^{1\cdot 3}x^{3\cdot 3}y^{2\cdot 3}$	Rules 2 and 3
$V = \frac{4}{3}\pi \cdot 2^3x^9y^6$	
$V = \frac{4}{3} \cdot 8 \cdot \pi \cdot x^9y^6$	Commutative Property
$V = \frac{32}{3}\pi x^9y^6$ cm^3	

Checkpoint

Find the volume of a rectangular prism with a length of $8x^2y$, a width of $3xy^3$, and a height of $3xy$ units.

$V = lwh; l = 8x^2y, w = 3xy^3, h = 3xy$

$V = (8x^2y)(3xy^3)(3xy)$

$V = 8 \cdot 3 \cdot 3 \cdot x^{2+1+1}y^{1+3+1}$

$V = 72x^4y^5$ cubic units

Checkpoint

To continue past this checkpoint, students should confidently and correctly answer this problem.

Q: Should any of the exponents be multiplied together for this problem? Explain.

A: *No, since no exponents are being raised to a power, the exponents should be added, not multiplied.*

Q: Where can you find the formula for the volume of a rectangular prism?

A: *The Formula Sheet.*

Q: Why are the units to a volume problem written in cubic units?

A: *Volume represents figures with length, width, and height (3D). When these three dimensions are multiplied to find the volume, you get a cubic unit.*

Worked solutions for these problems are located in the Digital Pack.

3)
Q: Are there different exponent rules when the expression has a rational exponent? Explain.
A: *No, the product and power rules are the same for integers and rational numbers.*

10)
Q: What do you need to do to the coefficients before multiplying them together?
A: *Make sure their exponents are equal to one.*

11)
Q: Which variable, a or b, do you need to look at to find the value of n?
A: *Since n is part of the exponent of a, you must look at a to find n.*

12–17)
Q: Before finding the volume or area, what should you do?
A: *Write down the correct formula and identify the known variables to substitute into the formula.*

Q: Why is using parentheses important when working with formulas?
A: *Parentheses are important because they remind you that all of the values substituted need to be raised to a power.*

Q: When you complete an area or volume formula, you should always check for correct __________.
A: *labels/units*

15) Your student may need to be reminded that "in terms of pi" means they can use the symbol for pi (π) in their answer instead of substituting the numeric value.

Practice 1

Complete practice problems on a separate sheet of paper.

Simplify. Assume all variables are positive.

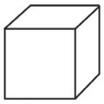
1) $\left(x^2y^3\right)^2 \cdot x^8$ $\mathbf{x^{12}y^6}$
2) $xy \cdot \left(xy^2\right)^3$ $\mathbf{x^4y^7}$
3) $\left(a^{12}b^6\right)^{\frac{1}{3}} \cdot a^2b^5$ $\mathbf{a^6b^7}$
4) $(xy)^4 \cdot \left(x^2y\right)^2$ $\mathbf{x^8y^6}$
5) $a^{\frac{1}{3}}b^{\frac{4}{3}} \cdot \left(ab^2\right)^{\frac{1}{3}}$ $\mathbf{a^{\frac{2}{3}}b^2}$
6) $\left(a^7b^3\right)^2 \cdot a^{-1}b^{-2}$ $\mathbf{a^{13}b^4}$
7) $2x \cdot \left(3x^5\right)^2$ $\mathbf{18x^{11}}$
8) $\left(3x^5y^7\right)^2 \cdot 5x$ $\mathbf{45x^{11}y^{14}}$
9) $(5xy)^2 \cdot \left(2x^2y\right)^3$ $\mathbf{200x^8y^5}$
10) $x^{-1}y^{-5} \cdot \left(x^2y^3z^4\right)^5$ $\mathbf{x^9y^{10}z^{20}}$

11) Find the value of n. $(5a)^2 \cdot 3a^nb^9 = 75a^7b^9$ $\mathbf{n = 5}$

For problems 12–17, choose the appropriate formula from the list below. Remember to label your answer with the proper units of measure.

Sphere	Rectangular prism	Rectangle	Square	Cylinder	Triangle	Cube
$V = \frac{4}{3}\pi r^3$	$V = lwh$	$A = lw$	$A = s^2$	$V = \pi r^2 h$	$A = \frac{1}{2}bh$	$V = s^3$

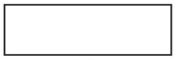
12) Find the volume of a cube with a side of $2x^3y$ feet.
$8x^9y^3$ ft^3



$2x^3y$ feet

13) Find the area of a triangle with a base of $6x$ units and a height of $2x^2$ units.
$6x^3$ square units

14) Find the area of a rectangle with a length of $8a^3b^2$ centimeters and a width of $7a^5b^4$ centimeters.
$56a^8b^6$ cm^2



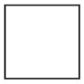
$7a^5b^4$ cm

$8a^3b^2$ cm

15) Find the volume of a cylinder in terms of pi when the height is $11x^3$ inches and the radius is x^5 inches.
$11\pi \cdot x^{18}$ in^3

16) Find the area of a rectangle with a width of ab^5 yards and a length of a^2c yards.
a^3b^5c yd^2

17) Find the area of a square with a side of $4x$ units.
$16x^2$ square units



$4x$ units

MASTERY CHECK 19B

Mastery Check

Show What You Know

The volume formula for a square pyramid is $V = \frac{1}{3}h \cdot s^2$, where h represents the height and s represents the length of the side of the base. The height of the pyramid is $3a^3$ inches. The length of a side of the base is $2ab^5$ inches.

A) Determine the volume of the pyramid.

$V = \frac{1}{3}h \cdot s^2$

$V = \frac{1}{3}(3a^3) \cdot (2ab^5)^2$

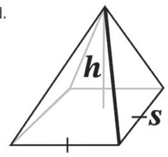
$V = \frac{3a^3}{3} \cdot 2^{1 \cdot 2} \cdot a^{1 \cdot 2} \cdot b^{5 \cdot 2}$

$V = a^3 \cdot 2^2 \cdot a^2 \cdot b^{10}$

$V = 2^2 \cdot a^{3+2} \cdot b^{10}$

$V = 4a^5b^{10}$ in^3

$4a^5b^{10}$ in^3



B) Another student *incorrectly* determined the volume to be $4a^3b^{10}$ cubic inches. Using the provided work, determine the error and explain the misunderstanding.

Step 1 $V = \frac{1}{3}(3a^3) \cdot (2ab^5)^2$

Step 2 $V = \frac{3a^3}{3} \cdot 2^{1 \cdot 2} \cdot a^{1 \cdot 2} \cdot b^{5 \cdot 2}$

Step 3 $V = a \cdot 2^2 \cdot a^2 \cdot b^{10}$

Step 4 $V = 2^2 \cdot a^{1+2} \cdot b^{10}$

Step 5 $V = 4a^3b^{10}$ in^3

Sample:
In Step 3, the student did not correctly simplify the first term. While three over three equals one $\left(\frac{3}{3} = 1\right)$, a raised to the power of three divided by three $\left(\frac{a^3}{3}\right)$ does not simplify the exponent to one. You cannot divide the exponent in the problem by 3. This student confused simplifying fractions with exponents.

C) Evaluate your answer from part A when $a = 2$ and $b = 1$.

$V = 4a^5b^{10}$

$V = (4)(2)^5(1)^{10}$

$V = 4 \cdot 32 \cdot 1$

$V = 128$ in^3

128 in^3

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

Show What You Know

A)

Q: What exponent rules are being used in this formula?

A: *Rules 1, 2, and 3.*

C)

Q: Why is it reasonable to determine the value of b even though it is raised to the 10th power?

A: *Because one to any power is still one.*

If your student does not have part A correct and solves this problem correctly using their expression, be sure to discuss their error in part A and how this would change the answer to part C.

Say What You Know

Your student should be able to restate the objectives of the lesson in their own words. If your student is unable to restate the lesson objectives, have them go back and reread the objectives and then explain them.

- Simplify exponential expressions using both the product and power rules for exponents.
- Apply the product and power rules for exponents to formulas.

Lesson Test

Is your student ready for the Lesson Test? After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

YES

If your student can answer "yes" to all of these questions, decide if your student is ready to take the Lesson Test.

NOT YET

If your student cannot answer "yes" to all of these questions, consider having your student complete some of these options:

- Rework Practice 1.
- Complete Practice 2
- Review the videos, Guided Notes, and Examples.

Worked solutions for these problems are located in the Digital Pack.

If needed, have your student go back to the Mastery Check and reapply what they have learned to say and show what they know.

19B PRACTICE 2

Practice 2

Complete practice problems on a separate sheet of paper.

Simplify. Assume all variables are positive.

1) $\left(x^{\frac{1}{3}}y^{6}\right)^{4} \cdot x^{\frac{1}{3}}y$ $\boldsymbol{x^{\frac{5}{3}}y^{25}}$

2) $(a^{3}b)^{\frac{1}{2}} \cdot a^{\frac{1}{2}}b$ $\boldsymbol{a^{2}b^{\frac{3}{2}}}$

3) $2^{3}ab \cdot (2a^{3}b)^{2}$ $\boldsymbol{2^{5}a^{7}b^{3}}$

4) $(3a^{4}b^{7})^{2} \cdot 3^{14}a^{6}b^{-2}$ $\boldsymbol{3^{16}a^{14}b^{12}}$

5) $(xy)^{3} \cdot (xy)^{6}$ $\boldsymbol{x^{9}y^{9}}$

6) $x^{3}y \cdot (x^{5}y^{3})^{3}$ $\boldsymbol{x^{18}y^{10}}$

7) $(3x^{2}y^{5})^{3} \cdot 3xy$ $\boldsymbol{3^{4}x^{7}y^{16}}$

8) $\frac{1}{2}x^{-8}y^{-1} \cdot (4x^{11}y)^{2}$ $\boldsymbol{8x^{14}y}$

9) $(3ab)^{2} \cdot (3a^{2b})$ $\boldsymbol{27a^{4}b^{3}}$

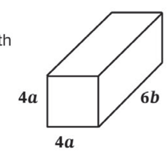
10) $(x^{8}y^{5})^{2} \cdot (2x^{5}y)^{3}$ $\boldsymbol{8x^{31}y^{13}}$

11) Find the value of n. $(a^{2}b^{3})^{5} \cdot (a^{8}c)^{n} = a^{26}b^{15}c^{2}$ $\boldsymbol{n = 2}$

For problems 12–17, choose the appropriate formula from the list below. Remember to label your answer with the proper units of measure.

Sphere	Rectangular prism	Rectangle	Square	Cylinder	Triangle	Cube
$V = \frac{4}{3}\pi r^{3}$	$V = lwh$	$A = lw$	$A = s^{2}$	$V = \pi r^{2}h$	$A = \frac{1}{2}bh$	$V = s^{3}$

12) Find the volume of a rectangular prism with a length and width of $4a$ feet and a height of $6b$ feet.
$\boldsymbol{V = 96a^{2}b}$ **ft**^{3}



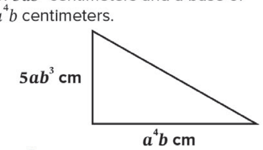
13) Find the volume of a sphere with a radius of $10x^{5}y^{3}$ units.
$\boldsymbol{V = \frac{4000}{3}\pi x^{15}y^{9}}$ **cubic units**

14) Find the area of a square with side lengths of $3ab$ inches.
$\boldsymbol{A = 9a^{2}b^{2}}$ **in**^{2}

15) Find the volume of a cube with sides of $5p^{7}q^{9}$ units.
$\boldsymbol{V = 125p^{21}q^{27}}$ **cubic units**

16) Find the area of a rectangle with a length of $3x^{11}$ feet and a width of $13x^{5}y^{2}$ feet.
$\boldsymbol{A = 39x^{16}y^{3}}$ **ft**^{2}

17) Find the area of a triangle with a height of $5ab^{8}$ centimeters and a base of $a^{4}b$ centimeters.



$\boldsymbol{A = \frac{5}{2}a^{5}b^{9}}$ **cm**^{2}

Lesson Test

Is your student ready for the Lesson Test? After achieving mastery for Parts A and B of this lesson, your student has the option to take the test. Before taking the test, ask your student these questions:

- Do you know all the new vocabulary words?
- Can you explain the objectives?
- Do you know how to check your work?
- Do you know how to use your Formula Sheet?
- Were you able to complete the practice questions without help?

YES

If your student can answer "yes" to all of these questions, decide if your student is ready to take the Lesson Test.

NOT YET

If your student cannot answer "yes" to all of these questions, consider having your student complete some of these options:

- Rework Practice 1.
- Complete Practice 2
- Review the videos, Guided Notes, and Examples.

TARGETED REVIEW 19

◎ Targeted Review

In the Targeted Review, you will practice topics you have mastered in earlier lessons. Reviewing these concepts will help you be successful as you work through this unit.

Problem	1	2–4	5	6	7	8	9–12	13	14	15–16	17	18
Lesson Origin	FS	1, 5	1	17, 18	V	2	1	17	2	PA	PA	7

Complete practice problems on a separate sheet of paper.

1) Find the perimeter of a square with sides of $(x + 8)$ feet using the formula $P = 4s$.
$P = 4x + 32$ ft

For problems 2–4, use problem 1 to answer the questions.

2) What algebraic property is used to find the perimeter? **The Distributive Property**

3) What would the perimeter be if $x = 4$? **$4(4) + 32 = 48$ ft**

4) Convert the perimeter from feet to yards. **$(48\text{ ft})\left(\frac{1\text{ yd}}{3\text{ ft}}\right) = 16\text{ yd}$**

5) Farmer McDonald is building a fence for their pigs. The total perimeter can be no more than 200 feet. The length must be at least 5 more than the width. Write a system of inequalities to represent fencing.

6) Write a system of equations. Do not solve. When the tens digit is subtracted from the units digit, the result is 5. The value of the digits is three times the sum of the digits.

7) What is the difference between an expression and an equation?

8) Write the equation. Do not solve. The variable x squared, plus twice x less six is zero.

For problems 9–12, classify the following as rational (Q) or irrational (I).

9) irrational + irrational = **irrational**

10) rational · irrational = **irrational**

11) rational + irrational = **irrational**

12) rational + rational = **rational**

13) Solve the system of equations. Name the method you used to solve.
$y = x + 1$ **(3, 4)**
$y = 2x - 2$ **Substitution**

14) Solve.
$6x - x + 3 = 4x + 7$
$x = 1$

For problems 15–16, find the factor pairs of the following numbers.

15) 100
1 and 100, 2 and 50, 4 and 25, 5 and 20, 10 and 10

16) 48
1 and 48, 2 and 24, 3 and 16, 4 and 12, 6 and 8

Worked solutions for these problems are located in the Digital Pack.

5) $2l + 2w \le 200$
$l \ge w + 5$

6) $u - t = 5$
$10t + u = 3(t + u)$

7) Sample: An equation uses an equals sign with an expression on either side. Equations can be solved.

Expressions do not have an equals sign but can be simplified, or evaluated, by combining like terms.

8) $x^2 + 2x - 6 = 0$

17) Distractor Rationale:
A) This would be the solution if place value is ignored.
C) This would be the solution if the numbers are not squared.
D) This would be the solution if the −4 is ignored.

18) Distractor Rationale:
A) This does not make the equation true when substituted.
B) These are the coefficients of the variables, not the solution.
D) These are the numbers in the given equation.

Multiple Choice

__B__ **17)** Evaluate the expression $-4ab^2 + a^2b$ when $a = 3$ and $b = 2$.

A) -400

B) -30

C) 30

D) 66

__C__ **18)** Sammi purchased p pencils and b books for school. Sammi spent a total of $26. The equation $p + 4b = 26$ represents the relationship between the number of books and pencils purchased. If the ordered pair $(6, 2)$ is the solution, what does this represent?

A) Sammi purchased 6 pencils and 2 books.

B) Sammi purchased 1 pencil and 4 books.

C) **Sammi purchased 6 books and 2 pencils.**

D) Sammi purchased 4 books and 26 pencils.