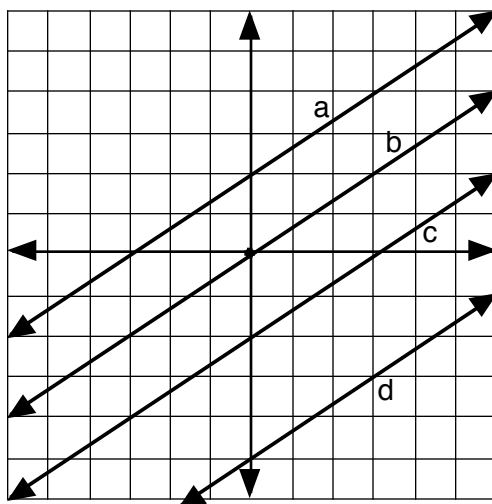


Lesson 8 Graphing Parallel Lines and the Standard Equation of a Line

Parallel Lines Two or more lines that have the same slope and different intercepts are parallel. Recall that parallel lines are defined as two lines in the same plane that never intersect or touch. We've talked about the fact that there are an infinite number of lines that have the same slope. The intercept distinguishes one from the other. In figure 1, notice that all the lines have a slope of $2/3$, and thus all are parallel. Only the intercepts are different for each line.

Figure 1



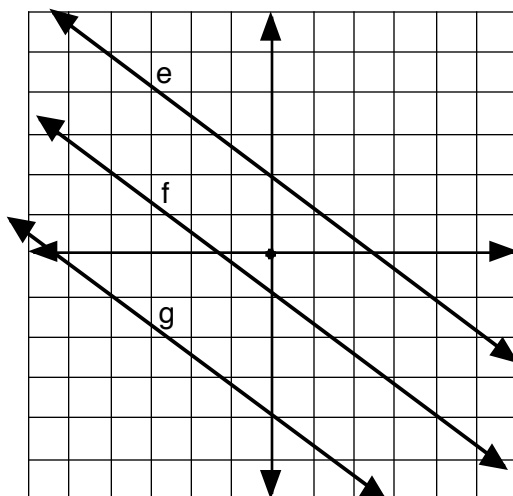
$$\text{line } a: Y = 2/3 X + 2$$

$$\text{line } b: Y = 2/3 X + 1, \text{ or } Y = 2/3 X$$

$$\text{line } c: Y = 2/3 X - 2$$

$$\text{line } d: Y = 2/3 X - 5$$

Figure 2



$$\text{line } e: Y = -3/4 X + 2$$

$$\text{line } f: Y = -3/4 X - 1$$

$$\text{line } g: Y = -3/4 X - 4$$

Equation of a Line As I mentioned in the DVD, another way of describing a line besides the slope-intercept formula, or form, is the standard form of the equation of a line. (Sometimes this is referred to simply as the “equation of a line,” or “standard form.”) Instead of $Y = mX + b$, it is defined as $AX + BY = C$. In the slope-intercept form, we want the coefficient of Y to be 1 (and thus not seen). In the standard form of the equation of a line, the coefficient of X is A (instead of m), the coefficient of Y is B (instead of 1), and both variables are written on the left-hand side of the equation.

While the slope-intercept form may have fractional coefficients, we will leave our standard form with whole number coefficients. Look at the next few examples, and notice the differences and similarities.

Example 1 Change the slope-intercept form to the standard form of the equation of a line.

$Y = 4/5 X + 2$ slope-intercept form, where the coefficient of Y is 1, the coefficient of X is 4/5, and b = 2.

$5Y = 4X + 10$ multiplying both sides by 5

$-4X + 5Y = 10$ subtracting 4X from both sides $A = -4, B = 5, C = 10$

$4X - 5Y = -10$ multiplying by -1 to make the coefficient of X positive $A = 4, B = -5, C = -10$

In example 1, either of the last two lines is correct as long as equation is in the form $AX + BY = C$.

Example 2 Change the slope-intercept form to the standard form of the equation of a line.

$Y = -2X - 1$ slope-intercept form; the coefficient of Y is 1, the coefficient of X is -2 and b = -1.

$2X + Y = -1$ adding 2X to both sides $A = 2, B = 1, C = -1$

Example 3 Change the slope-intercept form to the standard form of the equation of a line.

$Y = 5/3 X$ slope-intercept form; the coefficient of Y is 1, the coefficient of X is 5/3 and b = 0.

$3Y = 5X$ multiplying both sides by 3

$-5X + 3Y = 0$ subtracting 5X from both sides $A = -5, B = 3, C = 0$

Example 4 Change the standard form of the equation of a line to the slope-intercept form.

$2X + 3Y = 6$ standard equation of a line In the standard form, $A = 2, B = 3, C = 6$ $\begin{matrix} AX + BY = C \\ 2 \quad 3 \quad 6 \end{matrix}$

$3Y = -2X + 6$ subtracting 2X from both sides

$Y = -2/3 X + 2$ dividing both sides by 3 In the slope-intercept form, the coefficient of Y is 1, the coefficient of X is -2/3, and the intercept, or b, is 2.

Example 5 Change the standard form of the equation of a line to the slope-intercept form.

$-X - 2Y = 4$ standard equation of a line In the standard form, $A = -1, B = -2, C = 4$ $\begin{matrix} AX + BY = C \\ -1 \quad -2 \quad 4 \end{matrix}$

$-2Y = X + 4$ adding X to both sides

$Y = -1/2 X - 2$ dividing both sides by -2 In the slope-intercept form, the coefficient of Y is 1, the coefficient of X is -1/2, and the intercept, or b, is -2.

Example 6 Change the standard form of the equation of a line to the slope-intercept form.

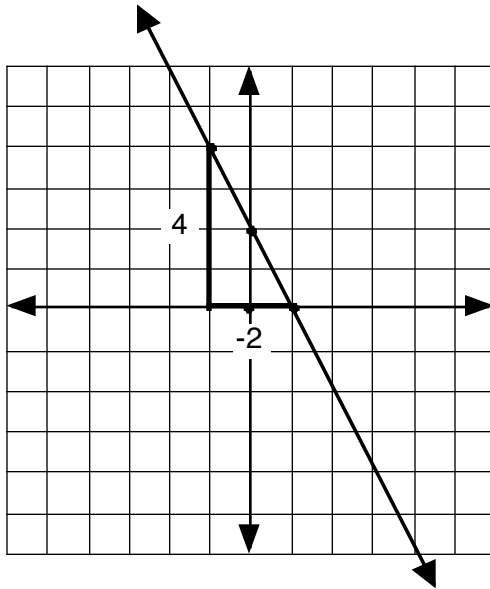
$-3X + 4Y = -8$ standard equation of a line In the standard form, $A = -3, B = 4, C = -8$ $\begin{matrix} AX + BY = C \\ -3 \quad 4 \quad -8 \end{matrix}$

$4Y = 3X - 8$ adding 3X to both sides

$Y = 3/4 X - 2$ dividing both sides by 4 In the slope-intercept form, the coefficient of Y is 1, the coefficient of X is 3/4, and the intercept, or b, is -2.

Writing an Equation Now let's put together all we've learned about plotting points, finding the slope, and describing the line with the slope-intercept form ($Y = mX + b$). Then we can describe a line using the standard form of an equation of a line ($AX + BY = C$).

Example 7 Plot the points $(-1, 4)$ and $(1, 0)$, find the slope intercept formula and the standard form of an equation of a line.



Estimating the slope of the line, we can see it will be negative since it is leaning backward.

Then we make a right triangle to find the slope.

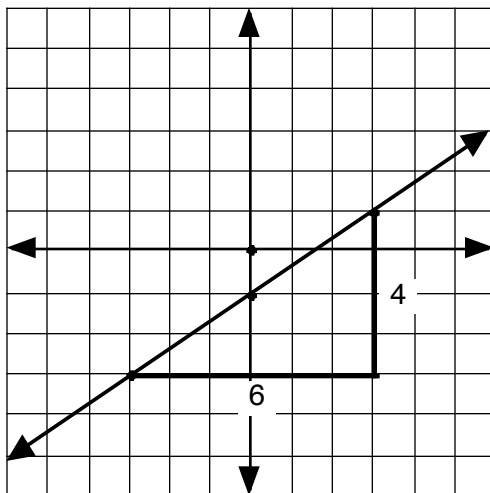
The rise (up distance) is 4, and the run is -2, since it is to the left. $M = \frac{\text{rise}}{\text{run}} = \frac{4}{-2} = -2$

Connecting the points reveals an intercept of +2, so $b = 2$.

The slope-intercept form is $Y = -2X + 2$.

The standard form is found by adding $+2X$ to both sides, giving us $2X + Y = 2$.

Example 8 Plot the points $(-3, -3)$ and $(3, 1)$, find the slope intercept formula and the standard form of an equation of a line.



Estimating the slope of the line, we can see it will be positive since it is falling forward.

Then we make a right triangle to find the slope.

The rise (up distance) is 4, and the run is 6, so the slope $m = \frac{\text{rise}}{\text{run}} = \frac{4}{6} = \frac{2}{3}$

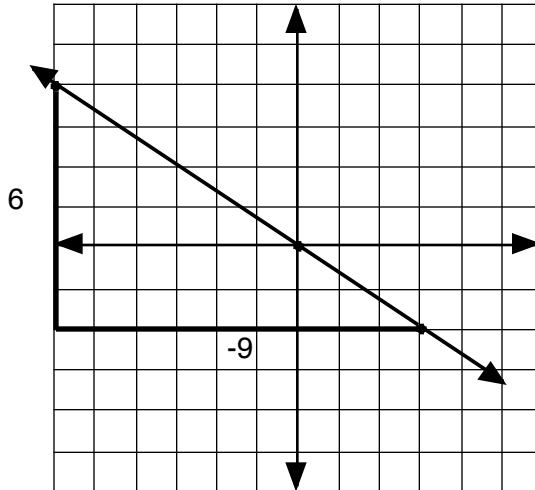
Connecting the points reveals an intercept of -1, so $b = -1$.

The slope-intercept form is $Y = 2/3X - 1$.

The standard form is found by subtracting $2/3X$ from both sides, and multiplying all the elements by 3: $3(-2/3X + Y = -1)$.

The standard form of the equation can be written as $-2X + 3Y = -3$, or $2X - 3Y = 3$

Example 9 Plot the points $(-6, 4)$ and $(3, -2)$, find the slope-intercept formula and the standard form of an equation of a line.



Estimating the slope of the line, we can see it will be negative since it is falling backward.

Then we make a right triangle to find the slope.

The rise is 6, and the run is -9 (to the left), so the

$$\text{slope } m = \frac{\text{rise}}{\text{run}} = \frac{6}{-9} = \frac{2}{-3}$$

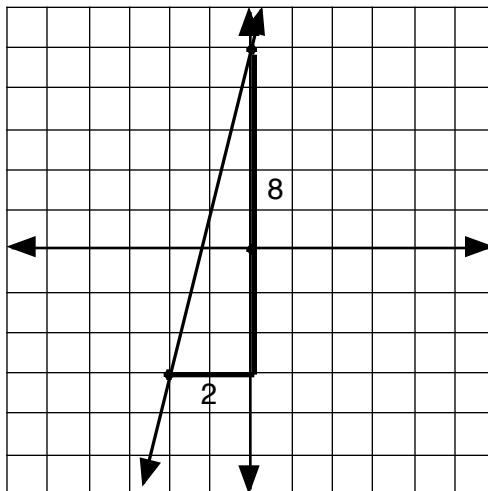
Connecting the points reveals an intercept of 0, so $b = 0$.

The slope-intercept form is $Y = -2/3 X + 0$, or $Y = -2/3 X$.

The standard form is found by adding $2/3 X$ to both sides, and multiplying all the elements by 3: $3(2/3 X + Y = 0)$.

The standard form of the equation is $2X + 3Y = 0$

Example 10 Plot the points $(0, 5)$ and $(-2, -3)$, find the slope intercept formula and the standard form of the equation of a line.



Estimating the slope of the line, we can see it will be positive since it is falling forward.

Then we make a right triangle to find the slope.

The rise (up distance) is 8, and the run is 2, so

$$\text{the slope } m = \frac{\text{rise}}{\text{run}} = \frac{8}{2} = \frac{4}{1} = 4$$

Connecting the points reveals an intercept of 5, so $b = 5$.

The slope-intercept form is $Y = 4X + 5$.

The standard form is found by subtracting $4X$ from both sides.

The standard form of the equation is $-4X + Y = 5$