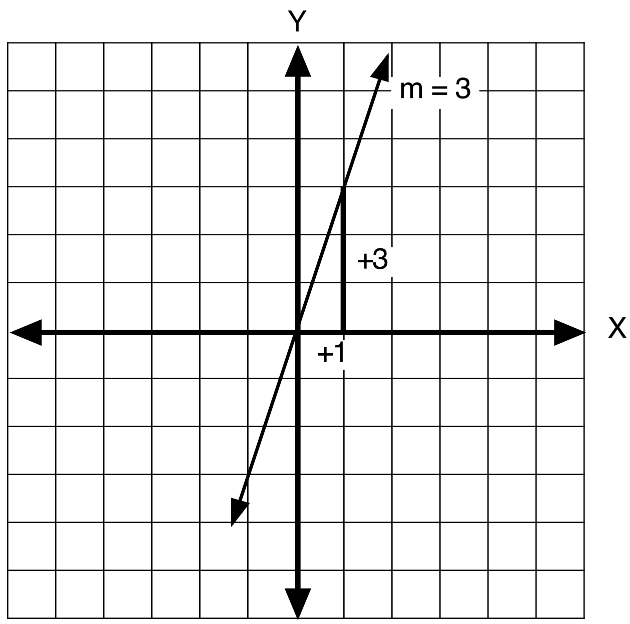
Lesson 7A Slope-Intercept Formula

Terms Two new words that describe what we've been doing in graphing lines are slope and intercept. The slope is referred to as "m" (a mountain has slope and starts with "m"), and intercept as "b" (a bee that intercepts your leg with his stinger!). The slope-intercept formula is Y = mX + b. In Y = 2X + 3, 2 is the slope and 3 is the intercept.

Slope Slope is the $\frac{\text{up dimension}}{\text{over dimension}}$, or the rise over the run ($\frac{\text{rise}}{\text{run}}$). In our example of the bread baking, for every one hour (over) we were able to bake three loaves (up). So for every hour we move over to the right one space and up three spaces. We continue to do this, and when we connect two or more points, we have a line that "slopes" up. We describe the slope as $\frac{\text{3 up}}{\text{1 over}}$ or 3.

Figure 1

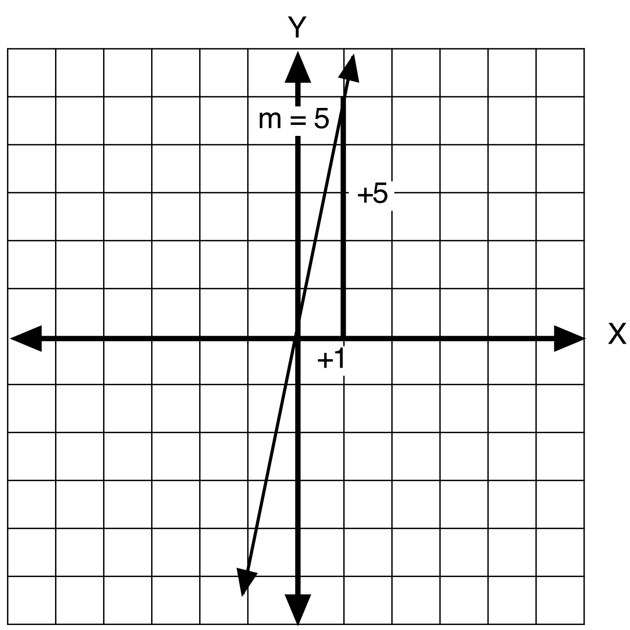


$\frac{\text{Rise}}{\text{Run}} = \frac{+3}{+1}$ = +3 or 3.

m = 3

If we were able to bake five loaves for every one hour we move over to the right one space and up five spaces. The slope is steeper. The rise is 5 and the run is 1. So the slope is $\frac{\text{5 up}}{\text{1 over}}$ or 5.

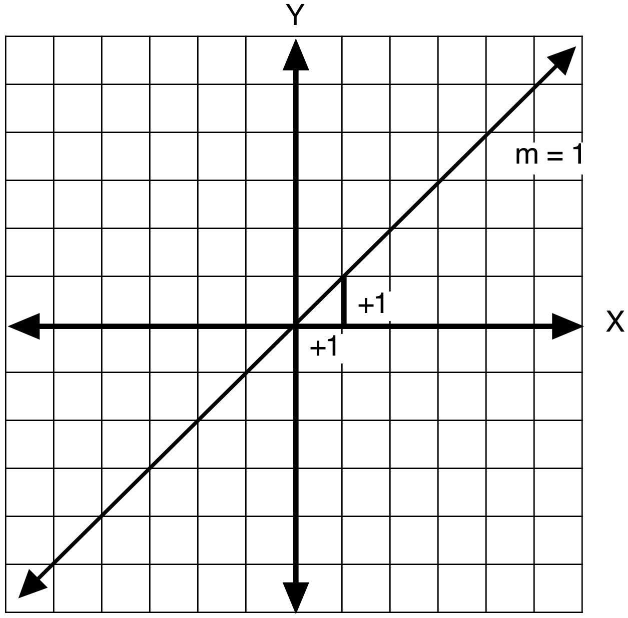
Figure 2



$\frac{\text{Rise}}{\text{Run}} = \frac{+5}{+1}$ = +5 or 5.

m = 5

Figure 3

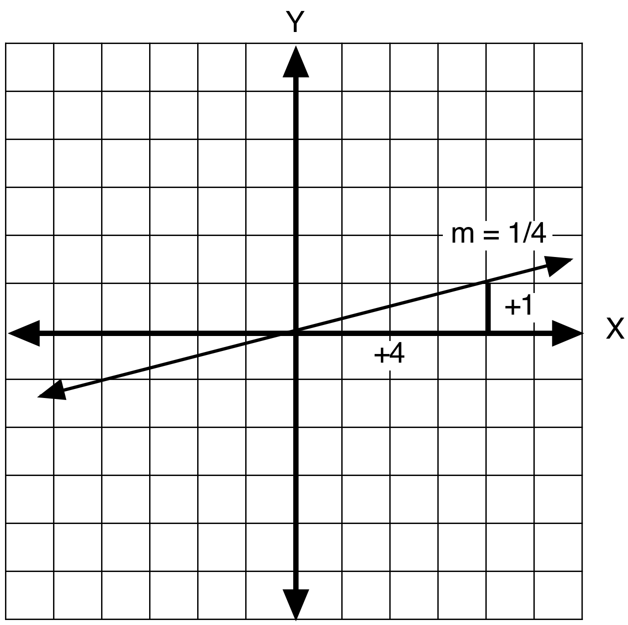


If we bake 1 loaf for every 1 hour, we move over to the right 1 space and up 1 space. The slope is not as steep as 3 loaves or 5 loaves per hour. The rise is 1 and the run is 1. The slope is $\frac{\text{1 up}}{\text{1 over}}$ or 1.

$$\frac{\text{Rise}}{\text{Run}} = \frac{+1}{+1} = +1 \text{ or } 1.$$

m = 1

Figure 4

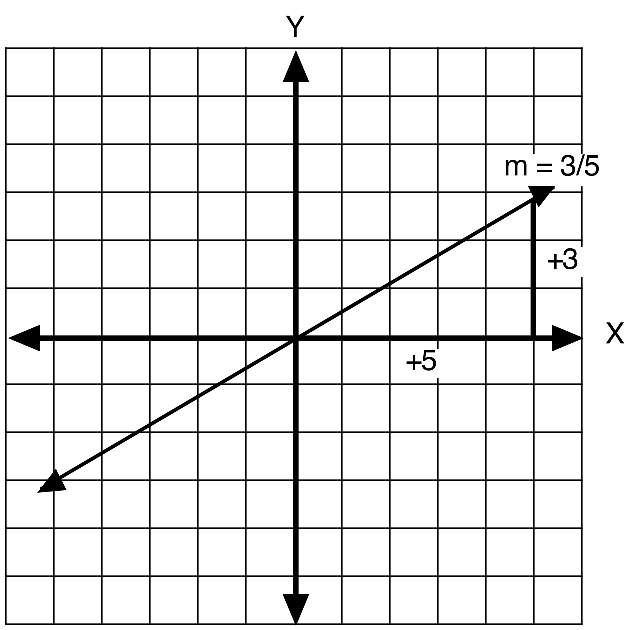


If we bake 1 loaf and it takes 4 hours, we move over to the right 4 spaces and up 1 space. The slope is not nearly as steep as 3 loaves per hour. The rise is 1 and the run is 4. The slope is $\frac{\text{1 up}}{\text{4 over}}$ or 1/4.

$$\frac{\text{Rise}}{\text{Run}} = \frac{+1}{+4} = +1/+4 \text{ or } 1/4.$$

m = 1/4

Figure 5



If we bake 3 loaves and it takes 5 hours, we move over to the right 5 spaces and up 3 spaces. The rise is 3 and the run is 5. The slope is $\frac{\text{3 up}}{\text{5 over}}$ or 3/5.

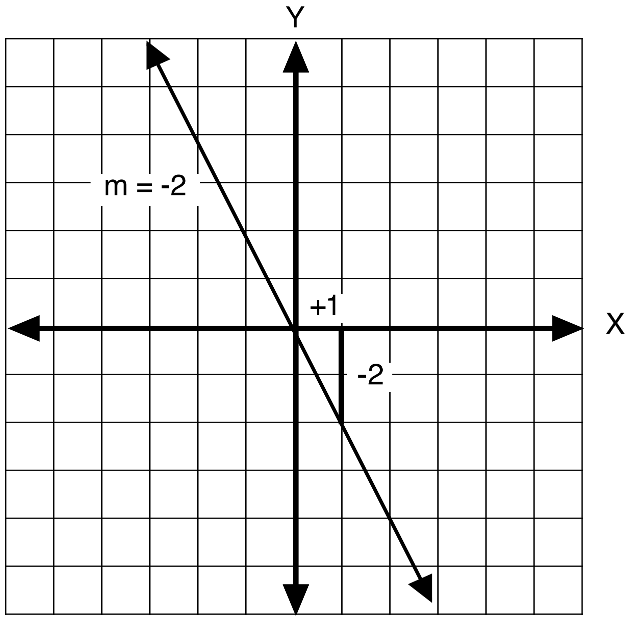
$$\frac{\text{Rise}}{\text{Run}} = \frac{+3}{+5} = +3/+5 \text{ or } 3/5.$$

m = 3/5

Negative Slope You can also have negative slopes. An example would be the business man who loses two dollars each day. The slope is over one and down two (the opposite of up, because it is minus). It will look like the line in figure 6: (Y = -2X + 0, or Y = -2X).

Days	Money
1	-2
2	-4
3	-6

Figure 6



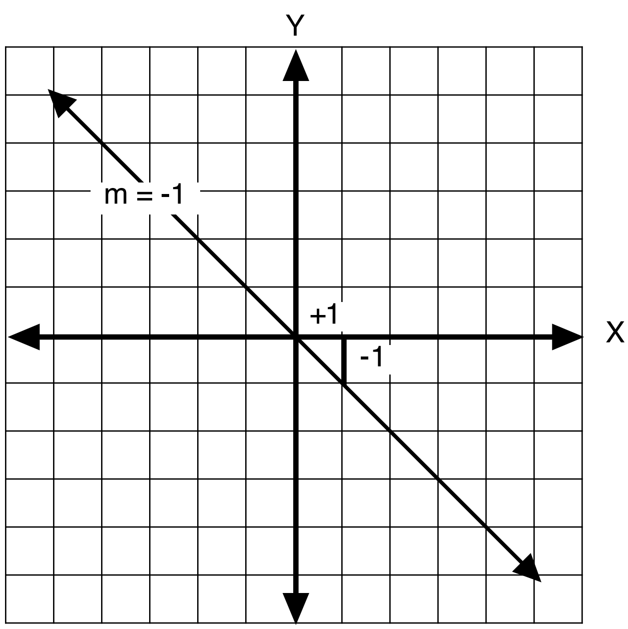
We move over to the right 1 space and up (really down) 2 spaces. The rise is -2 and the run is +1. The slope is $\frac{\text{2 down}}{\text{-1 over}}$ or -2/1 or -2.

$$\frac{\text{Rise}}{\text{Run}} = \frac{-2}{+1} = -2/+1 \text{ or } -2.$$

m = -2

Figure 7 is an example of losing one dollar per day: (Y = -X + 0, or Y = -X).

Figure 7



We move over to the right 1 space and up (really down) 1 space. The rise is -1 and the run is +1. The slope is $\frac{\text{1 down}}{\text{-1 over}}$ or -1/1 or -1. Notice that -1X is the same as -X.

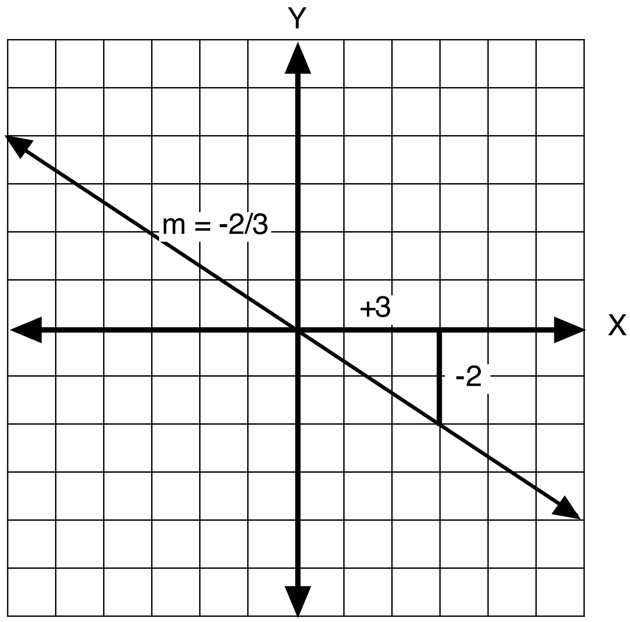
$$\frac{\text{Rise}}{\text{Run}} = \frac{-1}{+1} = -1/+1 \text{ or } -1.$$

m = -1

Negative Fraction Slope You can also have negative fraction slopes. An example would be the business man who loses two dollars every three days. The slope is over three and down two (the opposite of up, because it is minus).

Days	Money
3	-2
6	-4

Figure 8

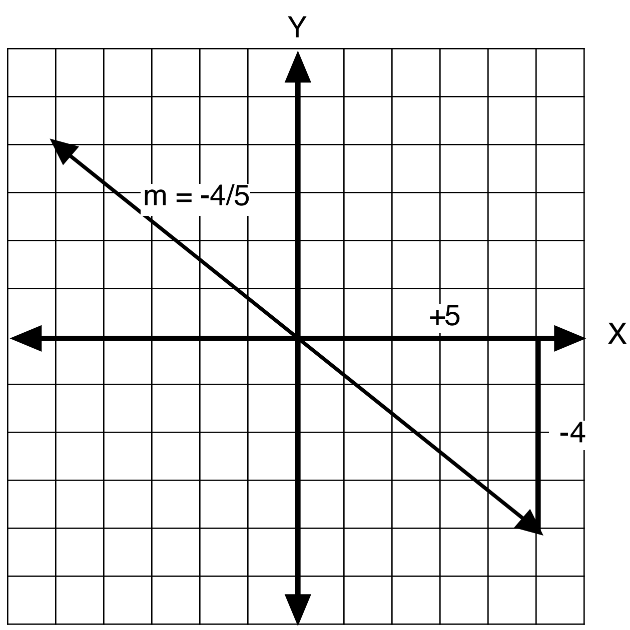


We move over to the right 3 spaces and up (really down) 2 spaces. The rise is -2 and the run is +3. The slope is $\frac{\text{2 down}}{\text{-3 over}}$ or -2/+3 or -2/3.

$$\frac{\text{Rise}}{\text{Run}} = \frac{-2}{+3} = -2/+3 \text{ or } -2/3.$$

m = -2/3

Figure 9



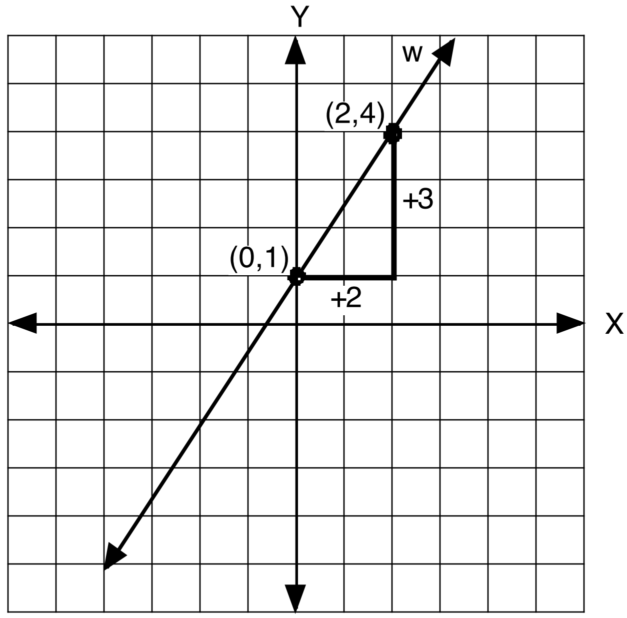
If the slope is -4/5, move over to the right 5 spaces and up (down) 4 spaces. The rise is -4 and the run is +5. The slope is $\frac{\text{4 down}}{\text{5 over}}$ or -4/+5 or -4/5.

$$\frac{\text{Rise}}{\text{Run}} = \frac{-4}{+5} = -4/+5 \text{ or } -4/5.$$

m = -4/5

Finding the Slope of a Line If you encounter the graph of a line, you can determine the slope by making a right triangle between any two points on the line.

Figure 10



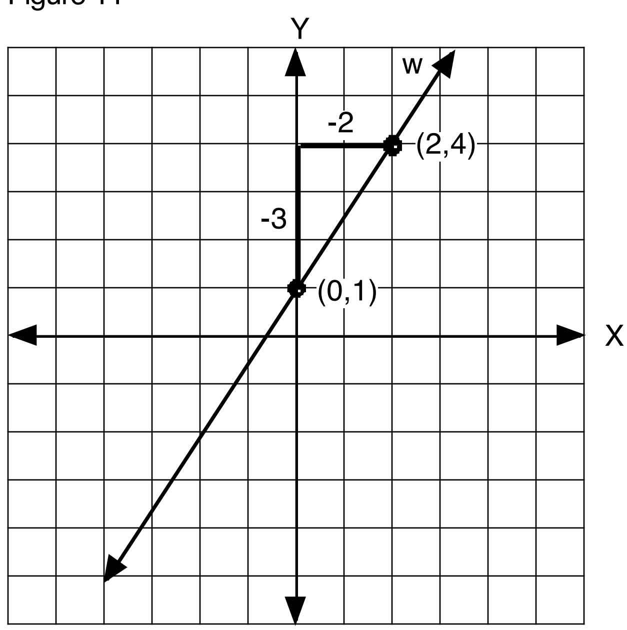
Here is how to find the slope of line *w* if it is not already given.

First choose two points that are on the grid, or at the intersection of a Y-coordinate and an X-coordinate. In figure 10, I chose two points: (2,4) and (0,1).

Then choose what direction you want to move. I chose to move away from the origin starting at the point (0,1) and moving towards (2,4). By making a right triangle, I can find the the up dimension or the rise, and the over dimension or the run. In figure 10 the rise is +3 and the run is +2.

The slope is +3/+2 (rise over run), or 3/2.

Figure 11



I could also have found the slope by moving towards the origin from point (2,4) towards (0,2).

In this case I drew the triangle above the line and moved to the left for the run, and down for the rise. Moving to the left on the Cartesian graph for the X-coordinates means the run will be a negative number. Moving down for the rise, or Y-coordinates, also yields a negative number. Both of these are negative values. In figure 11 the rise is -3 and the run is -2.

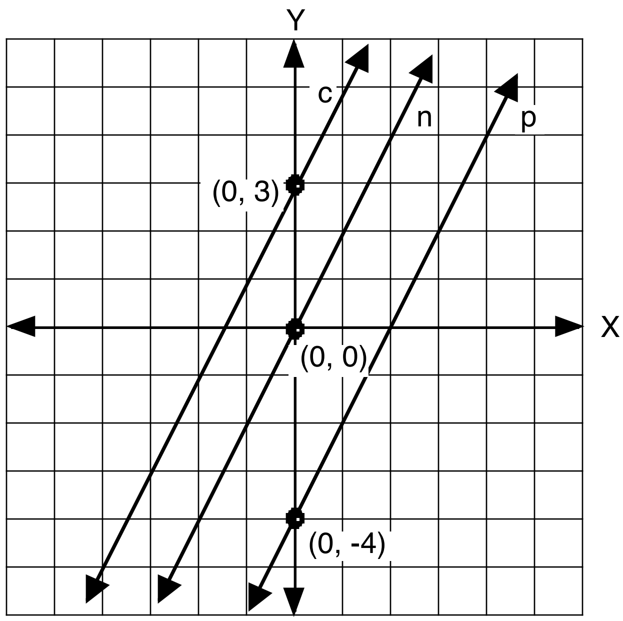
The slope is -3/-2 (rise over run), or 3/2.

So whether you move down the line or up the line, you still end up with a positive slope of 3/2.

Intercepts There are an infinite number of lines that have the same slope. Determining where a line crosses, or intercepts, the Y-axis narrows it down to one specific line.

In figure 12 there are 3 lines. Notice that each of these lines has the same slope. It is 2/1, or 2. What makes each line different is where it intercepts Y-axis (the Y-intercept).

Figure 12



The intercept of line *c* is the point (0, 3). All along the Y-axis, the coordinate for X is 0. The point (0, 3) tells us that X = 0 and Y = 3. So, our Y-intercept is found whenever X is 0, and in this case the intercept is 3.

Here is the intercept for each line:

line *c* (0, 3)
line *n* (0, 0)
line *p* (0, -4)

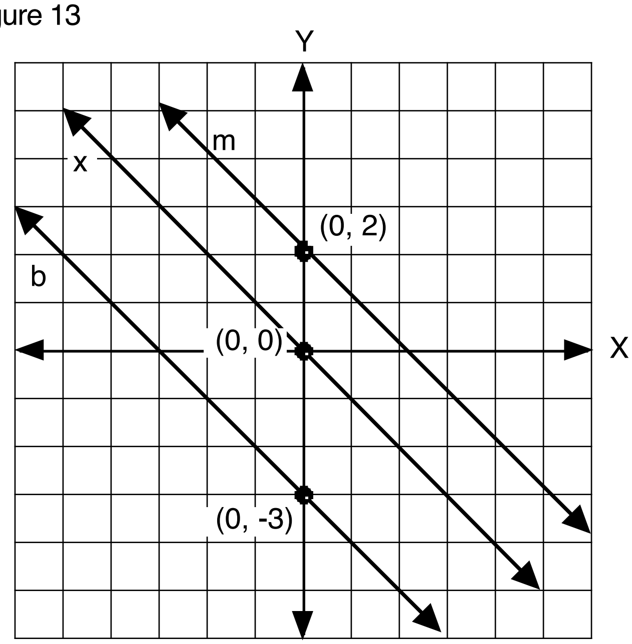
The slope of each line is 2, so the slope-intercept formulas for the lines are:

line *c*: Y = 2X + 3
line *n*: Y = 2X + 0 or Y = 2X
line *p*: Y = 2X - 4.

If an equation is given with no *b* term, we can assume that *b* is zero. This means that the Y-intercept of the line is also zero. See line *n* above.

We looked at the graph of each line, and it was easy to see the value of the intercept. If you have an equation of a line in the slope-intercept form, you can find the intercept algebraically by inserting “0” as the value of “X”, because when the line intercepts the Y-axis, X is always equal to 0. In line *c* above, Y = 2X + 3. If you insert the value 0 for X, then Y = 2(0) + 3, or Y = 3, and the intercept is 3. In line *p* above, Y = 2X - 4. When you insert the value 0 for X, then Y = 2(0) - 4, or Y = -4, and the intercept is -4.

Figure 13



The intercept of line *m* is the point (0, 2). All along the Y-axis, the coordinate for X is 0. The point (0, 2) tells us that when X = 0, then Y = 2. So, our Y-intercept is found whenever X is 0, and in this case the intercept is 2.

Here is the intercept for each line.

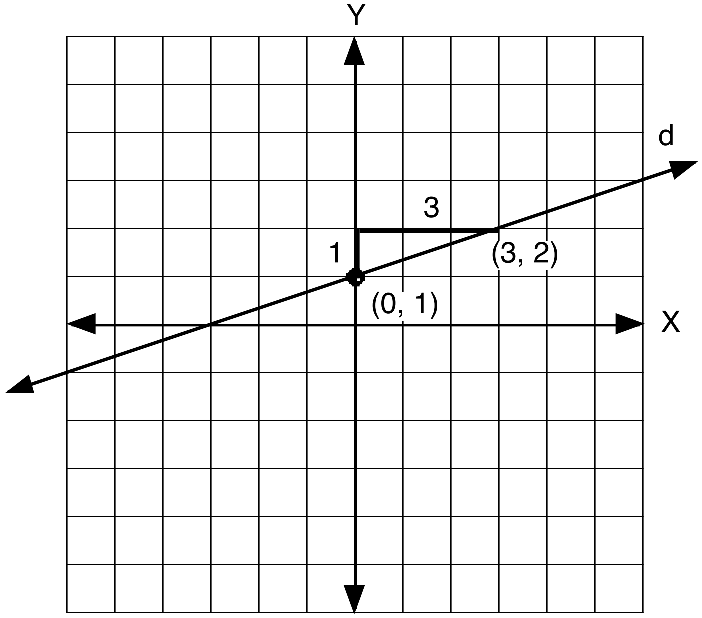
line *m* (0, 2)
line *x* (0, 0)
line *b* (0, -3)

The slope of each line is -1, so the slope-intercept formulas for the lines are:

line *c*: Y = -X + 2
line *n*: Y = -X + 0 or Y = -X
line *p*: Y = -X - 3.

Figure 14

Find the slope and intercept of line *d*.



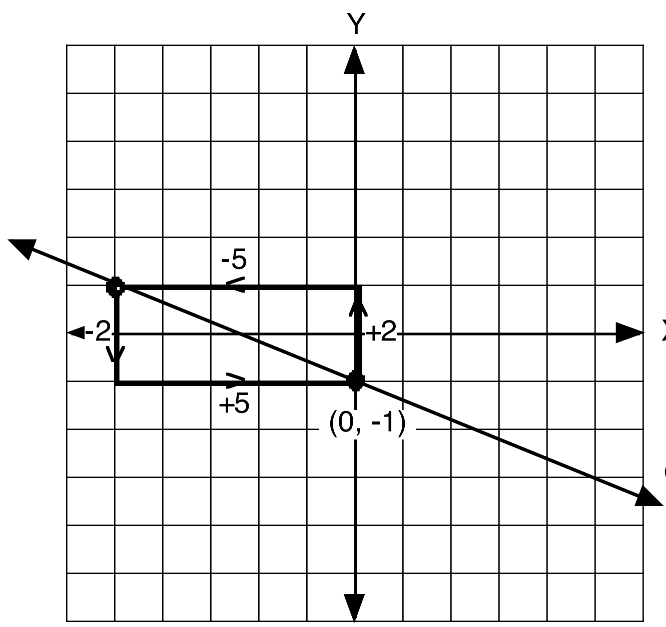
First I chose two points that are on the grid, or at the intersection of a Y-coordinate and an X-coordinate. In figure 14, I chose two points: (3,2) and (0,1).

I then chose to move away from the origin starting at the point (0,1) and moving towards (3,2). In making a right triangle, I found the up dimension, or the rise, to be +1, and the over dimension, or the run, to be +3. So the slope is +1/+3 (rise over run), or 1/3. Replacing *m* with 1/3, we get Y = 1/3 X.

I can see that the intercept is (0,1), so the slope-intercept formula representing line *d* is Y = 1/3 X + 1.

Figure 15

Find the slope and intercept of line *e*.



First I chose two points that are on the grid, or at the intersection of a Y-coordinate and an X-coordinate. In figure 14, I chose two points: (-5,1) and (0,-1).

I then chose to move away from the origin starting at the point (0,-1) and moving towards (-5,1). In making a right triangle, I found the up dimension, or the rise, to be +2, and the over dimension, or the run, to be -5. So the slope is +2/-5 (rise over run), or -2/5. Replacing *m* with -2/5 yields Y = -2/5 X.

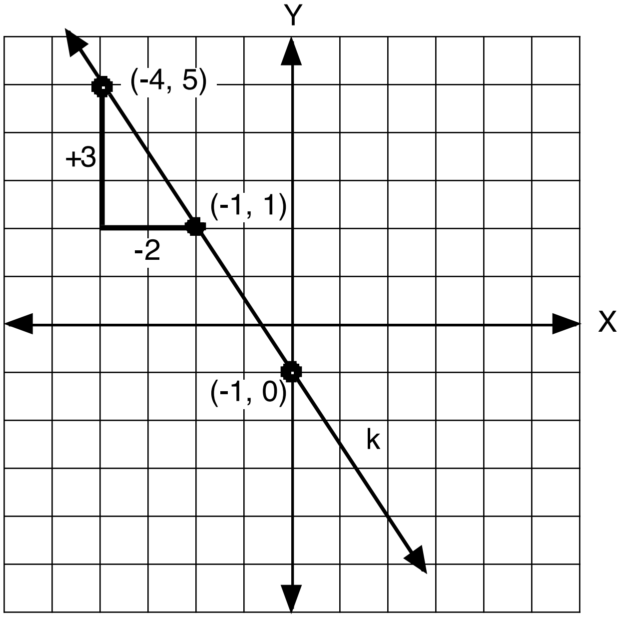
I can see that the intercept is (0,-1). So the slope-intercept formula representing line *e* is Y = -2/5 X - 1.

Notice that I could also have moved from (-5, 1) towards (0,-1) to find the slope. Then the rise would have been down 2, or -2, and the run would have been a positive 5. In that scenario the slope would have been -2 over +5, or -2/+5, which is equal to -2/5. Either way the slope is the same.

$$\frac{-2}{5} = \frac{2}{-5} = -\frac{2}{5}$$

Figure 16

Find the slope and intercept of line *k*.



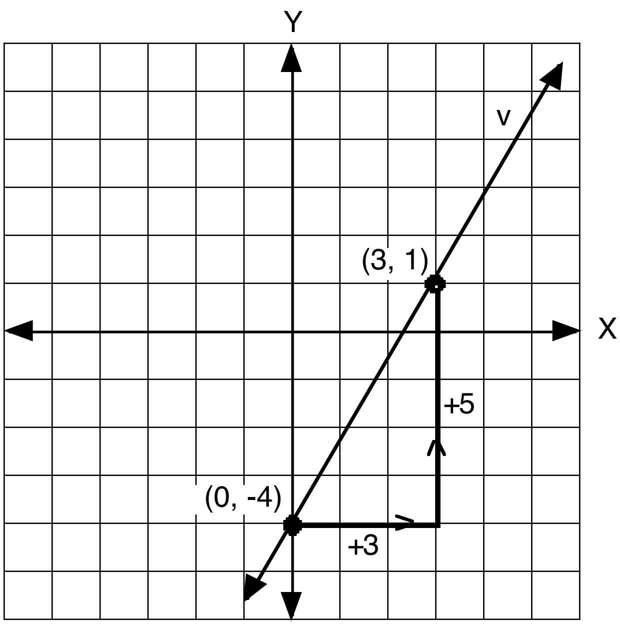
First I chose two points that are on the grid, or at the intersection of a Y-coordinate and an X-coordinate. In figure 16, I chose two points: (-2,2) and (-4,5).

I then chose to move away from the origin starting at the point (-2,2) and moving towards (-4,5). In making a right triangle, I found the up dimension, or the rise, to be +3, and the over dimension, or the run, to be -2. So the slope is +3/-2 (rise over run), or -3/2, and replacing *m* with -3/2 yields Y = -3/2 X.

I can see that the intercept is (0,-1). So the slope-intercept formula representing line *k* is
Y = -3/2 X - 1.

Figure 17

Find the slope and intercept of line *v*.



First I chose two points that are on the grid, or at the intersection of a Y-coordinate and an X-coordinate. In figure 17, I chose two points: (0,-4) and (3,1).

I then chose to move away from the point (0,-4) and move towards (3,1). In making a right triangle, I found the up dimension, or the rise, to be +5, and the over dimension, or the run, to be +3. So the slope is +5/+3 (rise over run), or 5/3, and replacing *m* with 5/3 yields Y = 5/3 X.

I can see that the intercept is (0,-4). So the slope-intercept formula representing line *v* is
Y = 5/3 X - 4.

Notice that I could also have moved from (3, 1) towards (0,-4) to find the slope. Then the rise would have been down 5, or -5, and the run would have been to the left 3, or a -3. In that scenario the slope would have been -5 over -3, or -5/-3, which is equal to 5/3. Either way the slope is the same.

$$\frac{+5}{+3} = \frac{-5}{-3} = \frac{5}{3}$$